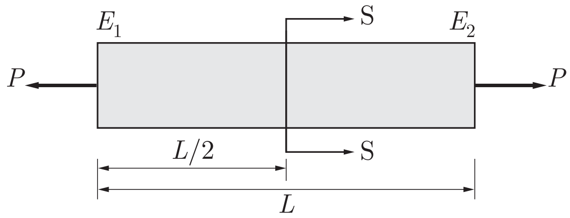
2 STRENGTH OF MATERIALS

YEAR 2013 — ONE MARK

MCQ 2.1 GATE ME 2013 ONE MARK

A rod of length L having uniform cross-sectional area A is subjected to a tensile force P as shown in the figure below. If the Young's modulus of the material varies linearly from E to E along the length of the rod, the normal stress developed at the section-SS is



(A) $\frac{P}{A}$ (B) $\frac{P(E - E)}{A(E + E)}$

(C) $\frac{PE}{AE}$ (D) $\frac{PE}{AE}$

MCQ 2.2

A long thin walled cylindrical shell, closed at both the ends, is subjected to an internal pressure. The ratio of the hoop stress (circumferential stress) to longitudinal stress developed in the shell is

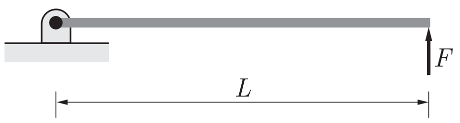
(A) 0.5 (B) 1.0

(C) 2.0 (D) 4.0

YEAR 2013 — TWO MARKS

MCQ 2.3 GATE ME 2013 TWO MARK

A pin joined uniform rigid rod of weight W and length L is supported horizontally by an external force F as shown in figure below. The force F is suddenly removed. At the instant of force removal, the magnitude of vertical reaction developed at the support is



(A) zero (B) W 4

(C) W 2 (D) W

MCQ 2.4

A simply supported beam of length L is subjected to a varying distributed load $\sin(3\pi x/L)$ Nm^{-1}, where the distance x is measured from the left support. The magnitude of the vertical reaction force in N at the left support is

(A) zero (B) L π

(C) L π (D) $2L/\pi$

MCQ 2.5

A bar is subjected to fluctuating tensile load from 20 kN to 100 kN. The material has yield strength of 240 MPa and endurance limit in reversed bending is 160 MPa. According to the Soderberg principle, the area of cross-section in mm^2 of the bar for a factor of safety of 2 is

(A) 400 (B) 600

(C) 750 (D) 1000

YEAR 2012 ONE MARK

MCQ 2.6 A thin walled spherical shell is subjected to an internal pressure. If the radius of the shell is increased by 1% and the thickness is reduced by 1%, with the internal pressure remaining the same, the percentage change in the circumferential (hoop) stress is

(A) 0 (B) 1

(C) 1.08 (D) 2.02

MCQ 2.7 A cantilever beam of length L is subjected to a moment M at the free end. The moment of inertia of the beam cross section about the neutral axis is I and the Young's modulus is E. The magnitude of the maximum deflection is

(A) $\frac{ML^2}{2EI}$ (B) $\frac{ML^2}{EI}$

(C) $\frac{2ML^2}{EI}$ (D) $\frac{4ML^2}{EI}$

MCQ 2.8 For a long slender column of uniform cross section, the ratio of critical buckling load for the case with both ends clamped to the case with both the ends hinged is

(A) 1 (B) 2

(C) 4 (D) 8

YEAR 2012 TWO MARKS

MCQ 2.9 The homogeneous state of stress for a metal part undergoing plastic deformation is

$$T = \begin{pmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{pmatrix}$$

where the stress component values are in MPa. Using Von Mises Yield criterion, the value of estimated shear yield stress, in MPa is

(A) 9.50 (B) 16.07

(C) 28.52 (D) 49.41

MCQ 2.10 The state of stress at a point under plane stress condition is

$$\sigma_{xx} = 40\text{ MPa},\ \sigma_{yy} = 100\text{ MPa and } \tau_{xy} = 40\text{ MPa}$$

The radius of the Mohr's circle representing the given state of stress in MPa is

(A) 40 (B) 50

(C) 60 (D) 100

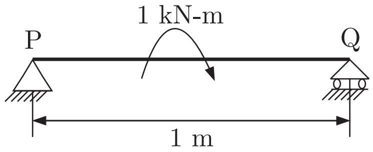
MCQ 2.11 A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is υ, the thermal stress developed in the cube due to heating is

(A) $-\frac{\alpha(\Delta T)E}{(1-2v)}$ (B) $-\frac{2\alpha(\Delta T)E}{(1-2v)}$

(C) $-\frac{3\alpha(\Delta T)E}{(1-2v)}$ (D) $-\frac{\alpha(\Delta T)E}{3(1-2v)}$

YEAR 2011 ONE MARK

MCQ 2.12 A simply supported beam PQ is loaded by a moment of 1 kNm at the mid-span of the beam as shown in the figure The reaction forces R_P and R_Q at supports P and Q respectively are



(A) 1 kN downward, 1 kN upward

(B) 0.5 kN upward, 0.5 kN downward

(C) 0.5 kN downward, 0.5 kN upward

(D) 1 kN upward, 1 kN upward

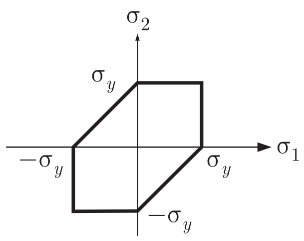
MCQ 2.13 A column has a rectangular cross-section of 10×20 mm and a length of 1 m. The slenderness ratio of the column is close to

(A) 200 (B) 346

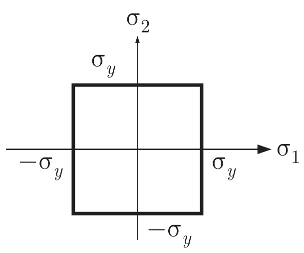
(C) 477 (D) 1000

MCQ 2.14 Match the following criteria of material failure, under biaxial stresses σ_1 and σ_2 and yield stress σ_y, with their corresponding graphic representations.

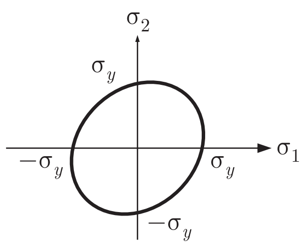
P. Maximum-normal-stress criterion L.



Q. Maximum-distortion-energy criterion M.



R. Maximum-shear-stress criterion N.



(A) P-M, Q-L, R-N (B) P-N, Q-M, R-L

(C) P-M, Q-N, R-L (D) P-N, Q-L, R-M

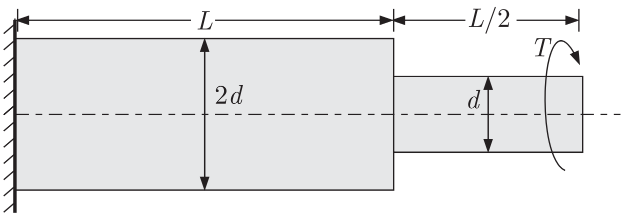
MCQ 2.15 A thin cylinder of inner radius 500 mm and thickness 10 mm is subjected to an internal pressure of 5 MPa. The average circumferential (hoop) stress in MPa is

(A) 100 (B) 250

(C) 500 (D) 1000

YEAR 2011 TWO MARKS

MCQ 2.16 A torque T is applied at the free end of a stepped rod of circular cross-section as shown in the figure. The shear modulus of material of the rod is G. The expression for d to produce an angular twist θ at the free end is

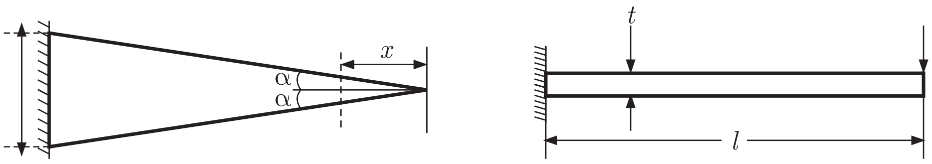


(A) $\left(\frac{2TL}{\pi\theta G}\right)^{\frac{1}{}}$ (B) $\left(\frac{1\ TL}{\pi\theta G}\right)^{\frac{1}{}}$

(C) $\left(\frac{1\ TL}{\pi\theta G}\right)^{\frac{1}{}}$ (D) $\left(\frac{2TL}{\pi\theta G}\right)^{\frac{1}{}}$

Common Data For Q. 12 and 13 :

A triangular-shaped cantilever beam of uniform-thickness is shown in the figure The Young's modulus of the material of the beam is E. A concentrated load P is applied at the free end of the beam.



MCQ 2.17 The area moment of inertia about the neutral axis of a cross-section at a distance x measured from the free end is

(A) $\frac{bxt^3}{6l}$ (B) $\frac{bxt^3}{12l}$

(C) $\frac{bxt^3}{24l}$ (D) $\frac{xt^3}{12l}$

MCQ 2.18 The maximum deflection of the beam is

(A) $\frac{2\ Pl}{Ebt}$ (B) $\frac{12Pl}{Ebt}$

(C) $\frac{Pl}{Ebt}$ (D) $\frac{Pl}{Ebt}$

YEAR 2010 ONE MARK

MCQ 2.19 The state of plane-stress at a point is given by $\sigma_x = -200$ MPa, $\sigma_y = 100$ MPa

$\tau_{xy} =$ a. The maximum shear stress (in MPa) is

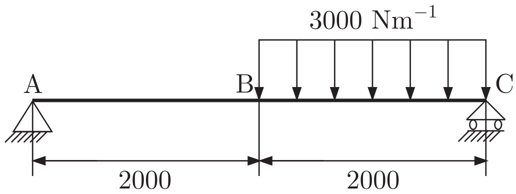
(A) 111.8 (B) 150.1

(C) 180.3 (D) 223.6

YEAR 2010 TWO MARKS

Common Data For Q.15 and Q.16

A massless beam has a loading pattern as shown in the figure. The beam is of rectangular cross-section with a width of 30 mm and height of 100 mm



MCQ 2.20 The maximum bending moment occurs at

(A) Location B (B) 2675 mm to the right of A

(C) 2500 mm to the right of A (D) 3225 mm to the right of A

MCQ 2.21 The maximum magnitude of bending stress (in MPa) is given by

(A) 60.0 (B) 67.5

(C) 200.0 (D) 225.0

YEAR 2009 ONE MARK

MCQ 2.22 If the principal stresses in a plane stress problem are $\sigma_1 = 100$ MPa, $\sigma_2 = 40$ MPa, the magnitude of the maximum shear stress (in MPa) will be

(A) 60 (B) 50

(C) 30 (D) 20

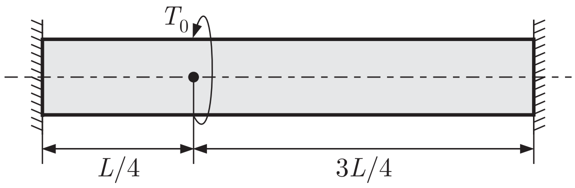
MCQ 2.23 A solid circular shaft of diameter d is subjected to a combined bending moment M and torque, T. The material property to be used for designing the shaft using the relation $\frac{16}{\pi d^3}\sqrt{M^2 + T^2}$ is

(A) ultimate tensile strength (S_u) (B) tensile yield strength (S_y)

(C) torsional yield strength (S_{sy}) (D) endurance strength (S_e)

YEAR 2009 TWO MARKS

MCQ 2.24 A solid shaft of diameter d and length L is fixed at both the ends. A torque, T_0 is applied at a distance $\frac{L}{4}$ from the left end as shown in the figure given below.

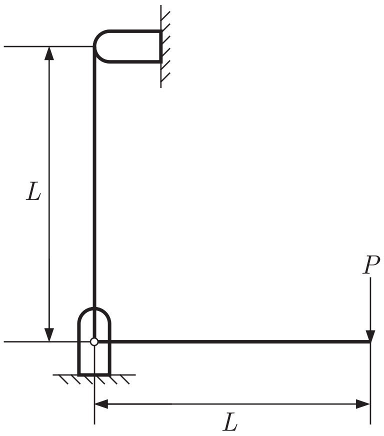


The maximum shear stress in the shaft is

(A) $\frac{T}{\pi d}$ (B) $\frac{T}{\pi d}$

(C) $\frac{T}{\pi d}$ (D) $\frac{T}{\pi d}$

MCQ 2.25 A frame of two arms of equal length L is shown in the adjacent figure. The flexural rigidity of each arm of the frame is EI. The vertical deflection at the point of application of load P is



(A) $\frac{PL^3}{3EI}$ (B) $\frac{2PL^3}{3EI}$

(C) $\frac{PL}{EI}$ (D) $\frac{4PL^3}{3EI}$

YEAR 2008 ONE MARK

MCQ 2.26 The transverse shear stress acting in a beam of rectangular cross-section, subjected to a transverse shear load, is

(A) variable with maximum at the bottom of the beam

(B) variable with maximum at the top of the beam

(C) uniform

(D) variable with maximum on the neutral axis

MCQ 2.27 A rod of length L and diameter D is subjected to a tensile load P. Which of the following is sufficient to calculate the resulting change in diameter ?

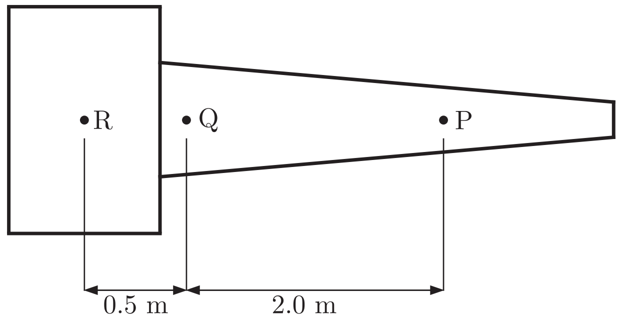
(A) Young's modulus

(B) Shear modulus

(C) Poisson's ratio

(D) Both Young's modulus and shear modulus

MCQ 2.28 A cantilever type gate hinged at Q is shown in the figure. P and R are the centers of gravity of the cantilever part and the counterweight respectively. The mass of the cantilever part is 75 kg. The mass of the counter weight, for static balance, is



(A) 75 kg (B) 150 kg

(C) 225 kg (D) 300 kg

MCQ 2.29 An axial residual compressive stress due to a manufacturing process is present on the outer surface of a rotating shaft subjected to bending. Under a given bending load, the fatigue life of the shaft in the presence of the residual compressive stress is

(A) decreased

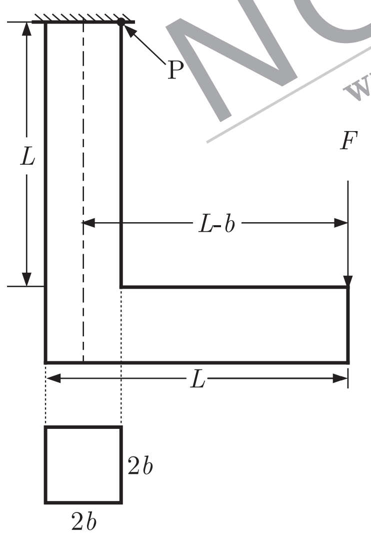
(B) increased or decreased, depending on the external bending load

(C) neither decreased nor increased

(D) increased

YEAR 2008 TWO MARKS

MCQ 2.30 For the component loaded with a force F as shown in the figure, the axial stress at the corner point P is



(A) $\frac{F(3L-b)}{4b^3}$ (B) $\frac{3(3L+b)}{4b^3}$

(C) $\frac{F(3L-4b)}{4b^3}$ (D) $\frac{F(3L-2b)}{4b^3}$

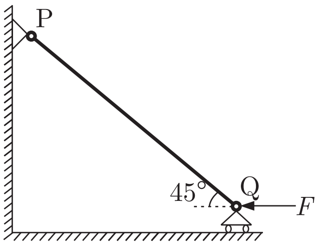
MCQ 2.31 A solid circular shaft of diameter 100 mm is subjected to an axial stress of 50 MPa. It is further subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is closest to

(A) 41 MPa (B) 82 MPa

(C) 164 MPa (D) 204 MPa

MCQ 2.32 The rod PQ of length L and with flexural rigidity EI is hinged at both ends.

For what minimum force F is it expected to buckle ?



(A) $\frac{\pi EI}{L}$ (B) $\frac{\sqrt{\ }\,\pi EI}{L}$

(C) $\frac{\pi^2 EI}{\sqrt{2}\,L^2}$ (D) $\frac{\pi^2 EI}{2L^2}$

MCQ 2.33 A compression spring is made of music wire of 2 mm diameter having a shear strength and shear modulus of 800 MPa and 80 GPa respectively. The mean coil diameter is 20 mm, free length is 40 mm and the number of active coils is 10. If the mean coil diameter is reduced to 10 mm, the stiffness of the spring is approximately

(A) decreased by 8 times (B) decreased by 2 times

(C) increased by 2 times (D) increased by 8 times

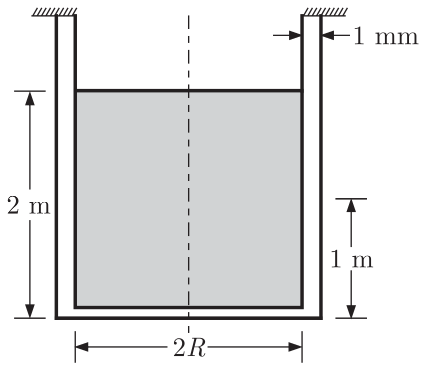
MCQ 2.34 A two dimensional fluid element rotates like a rigid body. At a point within the element, the pressure is 1 unit. Radius of the Mohr's circle, characterizing the state of stress at that point, is

(A) 0.5 unit (B) 0 unit

(C) 1 unit (D) 2 unit

Common Data For Q. 30 and 31 :

A cylindrical container of radius $R = 1$ m, wall thickness 1 mm is filled with water up to a depth of 2 m and suspended along its upper rim. The density of water is $1000\,\text{kg/m}^3$ and acceleration due to gravity is $10\,\text{m/s}^2$. The self-weight of the cylinder is negligible. The formula for hoop stress in a thin-walled cylinder can be used at all points along the height of the cylindrical container.



MCQ 2.35 The axial and circumference stress (σ_d, σ_c) experienced by the cylinder wall at mid-depth (1 m as shown) are

(A) (10, 10)MPa (B) (5, 10)MPa

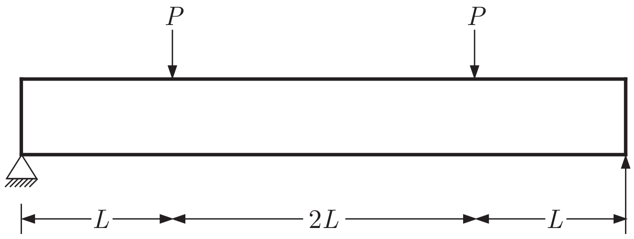
(C) (10, 5)MPa (D) (5, 5)MPa

MCQ 2.36 If the Young's modulus and Poisson's ratio of the container material are 100 GPa and 0.3, respectively, the axial strain in the cylinder wall at mid-depth is

(A) 2×10^{-5} (B) 6×10^{-5}

(C) 7×10^{-5} (D) 1.2×10^{-4}

MCQ 2.37 The strain energy stored in the beam with flexural rigidity EI and loaded as shown in the figure is

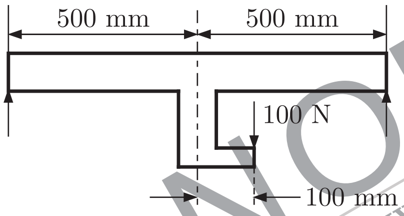


(A) $\frac{P^2L^3}{3EI}$ (B) $\frac{2P^2L^3}{3EI}$

(C) $\frac{4P^2L^3}{3EI}$ (D) $\frac{8P^2L^3}{3EI}$

YEAR 2007 ONE MARK

MCQ 2.38 In a simply-supported beam loaded as shown below, the maximum bending moment in Nm is



(A) 25 (B) 30

(C) 35 (D) 60

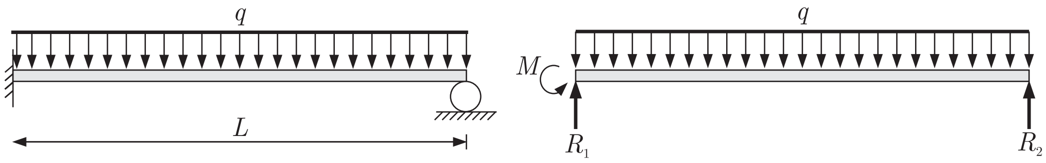
MCQ 2.39 A steel rod of length L and diameter D, fixed at both ends, is uniformly heated to a temperature rise of ΔT. The Young's modulus is E and the co-efficient of linear expansion is α. The thermal stress in the rod is

(A) 0 (B) $\alpha \Delta T$

(C) $E\alpha \Delta T$ (D) $E\alpha \Delta TL$

YEAR 2007 TWO MARKS

MCQ 2.40 A uniformly loaded propped cantilever beam and its free body diagram are shown below. The reactions are



(A) $R = \frac{5qL}{}, R_2 = \frac{3qL}{}, M = \frac{qL^2}{}$ (B) $R = \frac{3qL}{}, R_2 = \frac{5qL}{}, M = \frac{qL^2}{}$

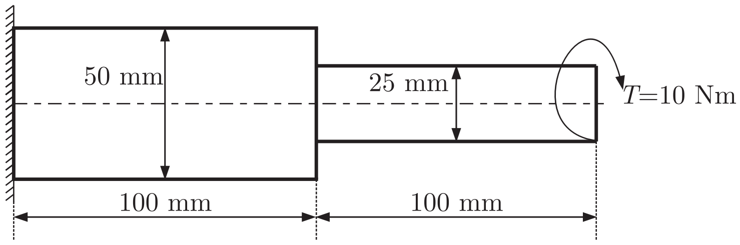
(C) $R = \frac{5qL}{}, R_2 = \frac{3qL}{}, M = 0$ (D) $R = \frac{3qL}{}, R_2 = \frac{5qL}{}, M = 0$

MCQ 2.41 A 200 × 100 × 50 mm steel block is subjected to a hydrostatic pressure of 15 MPa. The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3 respectively. The change in the volume of the block in mm^3 is

(A) 85 (B) 90

(C) 100 (D) 110

MCQ 2.42 A stepped steel shaft shown below is subjected to 10 Nm torque. If the modulus of rigidity is 80 GPa, the strain energy in the shaft in N-mm is

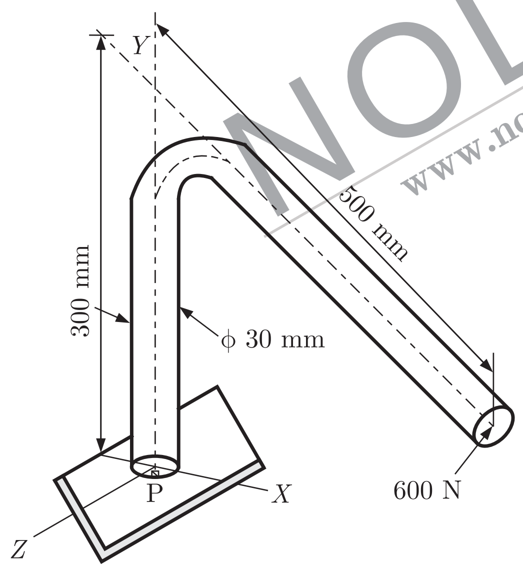


(A) 4.12 (B) 3.46

(C) 1.73 (D) 0.86

Common Data For Q. 38 and 39 :

A machine frame shown in the figure below is subjected to a horizontal force of 600 N parallel to Z-direction.



MCQ 2.43 The normal and shear stresses in MPa at point P are respectively

(A) 67.9 and 56.6 (B) 56.6 and 67.9

(C) 67.9 and 0.0 (D) 0.0 and 56.6

MCQ 2.44 The maximum principal stress in MPa and the orientation of the corresponding principal plane in degrees are respectively

(A) −32.0 and −29.52 (B) 100.0 and 60.48

(C) −32.0 and 60.48 (D) 100.0 and −29.52

YEAR 2006 ONE MARK

MCQ 2.45 For a circular shaft of diameter d subjected to torque T, the maximum value of the shear stress is

(A) $\frac{T}{\pi d}$ (B) $\frac{T}{\pi d}$

(C) $\frac{T}{\pi d}$ (D) $\frac{T}{\pi d}$

MCQ 2.46 A pin-ended column of length L , modulus of elasticity E and second moment of the cross-sectional area is I loaded eccentrically by a compressive load P. The critical buckling load (P_{cr}) is given by

(A) $P_{cr} = \frac{EI}{\pi L}$ (B) $P_{cr} = \frac{\pi EI}{L}$

(C) $P_{cr} = \frac{\pi EI}{L}$ (D) $P_{cr} = \frac{\pi EI}{L}$

YEAR 2006 TWO MARKS

MCQ 2.47 According to Von-Mises' distortion energy theory, the distortion energy under three dimensional stress state is represented by

(A) $\frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\upsilon(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

(B) $\frac{1-2\upsilon}{6E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

(C) $\frac{1+\upsilon}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

(D) $\frac{1}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \upsilon(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)]$

MCQ 2.48 A steel bar of 40 mm $\times$ 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and $E = 200$ GPa, the elongation of the bar will be

(A) 1.25 mm (B) 2.70 mm

(C) 4.05 mm (D) 5.40 mm

MCQ 2.49 A bar having a cross-sectional area of 700 mm^2 is subjected to axial loads at the positions indicated. The value of stress in the segment QR is

(A) 40 MPa (B) 50 MPa

(C) 70 MPa (D) 120 MPa

Common Data For Q. 45 and Q. 46 :

A simply supported beam of span length 6 m and 75 mm diameter carries a uniformly distributed load of 1.5 kN/m

MCQ 2.50 What is the maximum value of bending moment ?

(A) 9 kN-m (B) 13.5 kN-m

(C) 81 kN-m (D) 125 kN-m

MCQ 2.51 What is the maximum value of bending stress ?

(A) 162.98 MPa (B) 325.95 MPa

(C) 625.95 MPa (D) 651.90 MPa

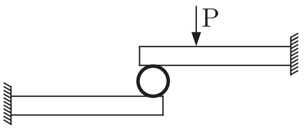
YEAR 2005 ONE MARK

MCQ 2.52 A uniform, slender cylindrical rod is made of a homogeneous and isotropic material. The rod rests on a frictionless surface. The rod is heated uniformly. If the radial and longitudinal thermal stresses are represented by σ_r and σ_z, respectively, then

(A) $\sigma_r = 0, \sigma_z = 0$ (B) $\sigma_r \neq 0, \sigma_z = 0$

(C) $\sigma_r = 0, \sigma_z \neq 0$ (D) $\sigma_r \neq 0, \sigma_z \neq 0$

MCQ 2.53 Two identical cantilever beams are supported as shown , with their free ends in contact through a rigid roller. After the load P is applied, the free ends will have



(A) equal deflections but not equal slopes

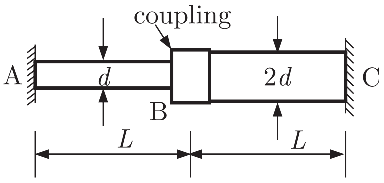
(B) equal slopes but not equal deflections

(C) equal slopes as well as equal deflections

(D) neither equal slopes nor equal deflections

YEAR 2005 TWO MARKS

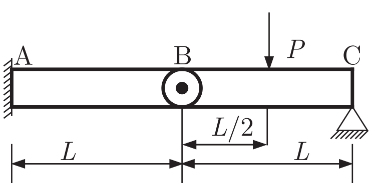
MCQ 2.54 The two shafts AB and BC, of equal length and diameters d and $2d$, are made of the same material. They are joined at B through a shaft coupling, while the ends A and C are built-in (cantilevered). A twisting moment T is applied to the coupling. If T_A and T_C represent the twisting moments at the ends A and C , respectively, then



(A) $T_C = T_A$ (B) $T_C = \quad T_A$

(C) $T_C = 16T_A$ (D) $T_A = 16T_C$

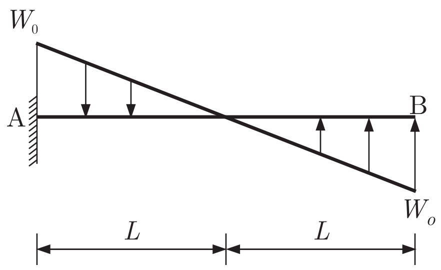
MCQ 2.55 A beam is made up of two identical bars AB and BC, by hinging them together at B. The end A is built-in (cantilevered) and the end C is simply-supported. With the load P acting as shown, the bending moment at A is

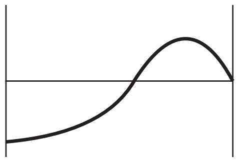
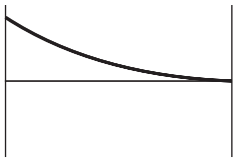


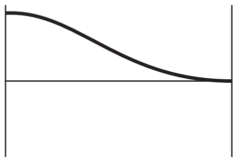
(A) zero (B) $\frac{PL}{2}$

(C) $\frac{3PL}{2}$ (D) indeterminate

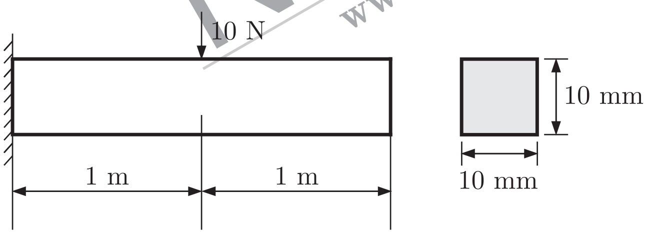
MCQ 2.56 A cantilever beam carries the anti-symmetric load shown, where W is the peak intensity of the distributed load. Qualitatively, the correct bending moment diagram for this beam is

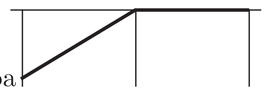


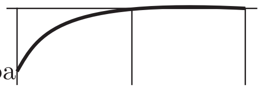
(A)  (B) 

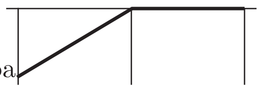
(C)  (D) 

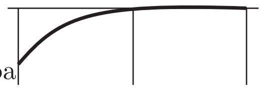
MCQ 2.57 A cantilever beam has the square cross section of 10 mm × 10 mm. It carries a transverse load of 10 N. Consider only the bottom fibres of the beam, the correct representation of the longitudinal variation of the bending stress is



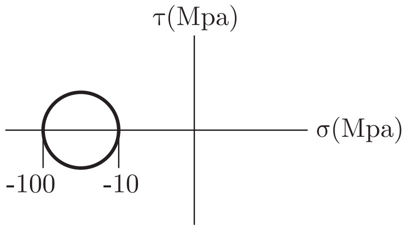
(A) 

(B) 

(C) 

(D) 

MCQ 2.58 The Mohr's circle of plane stress for a point in a body is shown. The design is to be done on the basis of the maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is



(A) 45 MPa
(B) 50 MPa
(C) 90 MPa
(D) 100 MPa

MCQ 2.59 A weighing machine consist of a 2 kg pan resting on a spring. In this condition, with the pan resting on the spring, the length of the spring is 200 mm. When a mass of 20 kg is placed on the pan, the length of the spring becomes 100 mm. For the spring, the un-deformed length L and the spring constant k (stiffness) are

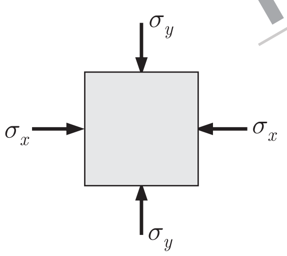
(A) $L = 220$ mm, $k = 1\ \ 2$ N/m
(B) $L = 210$ mm, $k = 19\ 0$ N/m
(C) $L = 200$ mm, $k = 19\ 0$ N/m
(D) $L = 200$ mm, $k = 215$ N/m

YEAR 2004 ONE MARK

MCQ 2.60 In terms of Poisson's ratio (v) the ratio of Young's Modulus (E) to Shear Modulus (G) of elastic materials is

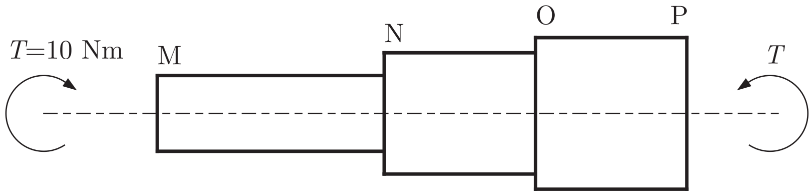
(A) $2(1+v)$
(B) $2(1-v)$
(C) $\frac{1}{2}(1+v)$
(D) $\frac{1}{2}(1-v)$

MCQ 2.61 The figure shows the state of stress at a certain point in a stressed body. The magnitudes of normal stresses in x and y directions are 100 MPa and 20 MPa respectively. The radius of Mohr's stress circle representing this state of stress is



(A) 120
(B) 80
(C) 60
(D) 40

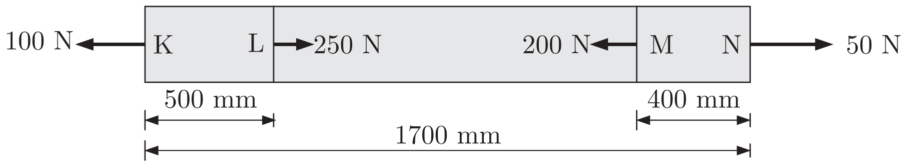
MCQ 2.62 A torque of 10 Nm is transmitted through a stepped shaft as shown in figure. The torsional stiffness of individual sections of length MN, NO and OP are 20 Nm/rad, 30 Nm/rad and 60 Nm/rad respectively. The angular deflection between the ends M and P of the shaft is



(A) 0.5 rad
(B) 1.0 rad
(C) 5.0 rad
(D) 10.0 rad

YEAR 2004 TWO MARKS

MCQ 2.63 The figure below shows a steel rod of 25 mm^2 cross sectional area. It is loaded at four points, K, L, M and N. Assume $E_{steel} =$ a. The total change in length of the rod due to loading is



(A) 1 μm (B) – 10 μm

(C) 16 μm (D) – 20 μm

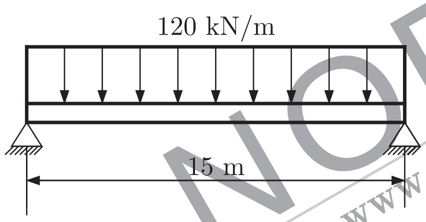
MCQ 2.64 A solid circular shaft of 60 mm diameter transmits a torque of 1600 N.m. The value of maximum shear stress developed is

(A) 37.72 MPa (B) 47.72 MPa

(C) 57.72 MPa (D) 67.72 MPa

Common Data For Q. 60 and 61 are given below.

A steel beam of breadth 120 mm and height 750 mm is loaded as shown in the figure. Assume $E_{steel} = 200$ Pa.



MCQ 2.65 The beam is subjected to a maximum bending moment of

(A) 3375 kN-m (B) 4750 kN-m

(C) 6750 kN-m (D) 8750 kN-m

MCQ 2.66 The value of maximum deflection of the beam is

(A) 93.75 mm (B) 83.75 mm

(C) 73.75 mm (D) 63.75 mm

YEAR 2003 ONE MARK

MCQ 2.67 The second moment of a circular area about the diameter is given by (D is the diameter).

(A) $\frac{\pi D^4}{4}$ (B) $\frac{\pi D^4}{16}$

(C) $\frac{\pi D^4}{32}$ (D) $\frac{\pi D^4}{64}$

MCQ 2.68 A concentrated load of P acts on a simply supported beam of span L at a distance L 3 from the left support. The bending moment at the point of application of the load is given by

(A) $\frac{PL}{3}$ (B) $\frac{2PL}{3}$

(C) $\frac{PL}{9}$ (D) $\frac{2PL}{9}$

MCQ 2.69 Two identical circular rods of same diameter and same length are subjected to same magnitude of axial tensile force. One of the rod is made out of mild steel having the modulus of elasticity of 206 GPa. The other rod is made out of cast iron having the modulus of elasticity of 100 GPa. Assume both the materials to be homogeneous and isotropic and the axial force causes the same amount of uniform stress in both the rods. The stresses developed are within the proportional limit of the respective materials. Which of the following observations is correct ?

(A) Both rods elongate by the same amount

(B) Mild steel rod elongates more than the cast iron rod

(C) Cast iron rod elongates more than the mild steel rods

(D) As the stresses are equal strains are also equal in both the rods

MCQ 2.70 The beams, one having square cross section and another circular cross-section, are subjected to the same amount of bending moment. If the cross sectional area as well as the material of both the beams are same then

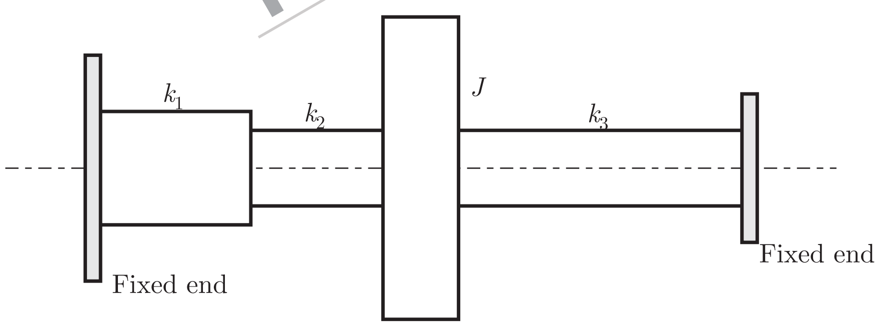
(A) maximum bending stress developed in both the beams is same

(B) the circular beam experience more bending stress than the square one

(C) the square beam experience more bending stress than the circular one

(D) as the material is same, both the beams will experience same deformation.

MCQ 2.71 Consider the arrangement shown in the figure below where J is the combined polar mass moment of inertia of the disc and the shafts. k_1, k_2, k are the torsional stiffness of the respective shafts. The natural frequency of torsional oscillation of the disc is given by



(A) $\sqrt{\frac{k_1+k_2+k}{J}}$ (B) $\sqrt{\frac{k_1k_2+k_2k+k\,k_1}{J(k_1+k_2)}}$

(C) $\sqrt{\frac{k_1+k_2+k}{J(k_1k_2+k_2k+k\,k_1)}}$ (D) $\sqrt{\frac{k_1k_2+k_2k+k\,k_1}{J(k_2+k)}}$

MCQ 2.72 Maximum shear stress developed on the surface of a solid circular shaft under pure torsion is 240 MPa. If the shaft diameter is doubled then the maximum shear stress developed corresponding to the same torque will be

(A) 120 MPa (B) 60 MPa

(C) 30 MPa (D) 15 MPa

YEAR 2003 TWO MARKS

MCQ 2.73 A simply supported laterally loaded beam was found to deflect more than a specified value. Which of the following measures will reduce the deflection ?

(A) Increase the area moment of inertia

(B) Increase the span of the beam

(C) Select a different material having lesser modulus of elasticity

(D) Magnitude of the load to be increased

MCQ 2.74 A shaft subjected to torsion experiences a pure shear stress τ on the surface. The maximum principal stress on the surface which is at 45° to the axis will have a value

(A) $\tau \cos 45^\circ$

(B) $2\tau \cos 45^\circ$

(C) $\tau \cos^2 45^\circ$

(D) $2\tau \sin 45^\circ \cos 45^\circ$

Common Data For Q. 70 and 71 are given below.

The state of stress at a point "P" in a two dimensional loading is such that the Mohr's circle is a point located at 175 MPa on the positive normal stress axis.

MCQ 2.75 The maximum and minimum principal stresses respectively from the Mohr's circle are

(A) $+175$ MPa, -175 MPa

(B) $+175$ MPa, $+175$ MPa

(C) 0, -175 MPa

(D) 0, 0

MCQ 2.76 The directions of maximum and minimum principal stresses at the point "P" from the Mohr's circle are

(A) 0, 90°

(B) 90°, 0

(C) 45°, 135°

(D) all directions

YEAR 2002 ONE MARK

MCQ 2.77 The total area under the stress-strain curve of mild steel specimen tested upto failure under tension is a measure of

(A) ductility

(B) ultimate strength

(C) stiffness

(D) toughness

MCQ 2.78 The number of components in a stress tensor defining stress at a point in three dimensions is

(A) 3

(B) 4

(C) 6

(D) 9

YEAR 2002 TWO MARKS

MCQ 2.79 The relationship between Young's modulus (E), Bulk modulus (K) and Poisson's ratio (υ) is given by

(A) $E = 3K(1 - \upsilon)$

(B) $K = 3E(1 - \upsilon)$

(C) $E = 3K(1 - \upsilon)$

(D) $K = 3E(1 - \upsilon)$

MCQ 2.80 The ratio of Euler's bucking loads of columns with the same parameters having

(i) both ends fixed, and (ii) both ends hinged is

(A) 2 (B) 4

(C) 6 (D) 8

YEAR 2001 ONE MARK

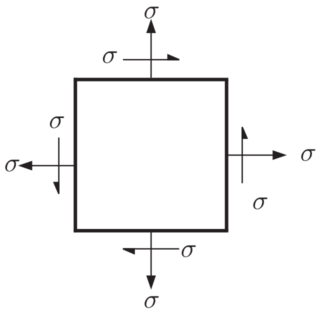
MCQ 2.81 The shape of the bending moment diagram for a uniform cantilever beam carrying a uniformly distributed load over its length is

(A) a straight line (B) a hyperbola

(C) an ellipse (D) a parabola

YEAR 2001 TWO MARKS

MCQ 2.82 The maximum principal stress for the stress state shown in the figure is



(A) σ (B) 2σ

(C) 3σ (D) 1.5σ

****

SOLUTION

SOL 2.82 Option (A) is correct.

The normal stress is given by

$$\sigma = \frac{P}{A}$$

We see that normal stress only depends on force and area and it does not depends on E.

SOL 2.82 Option (C) is correct.

Hoop stress or circumferential stress is

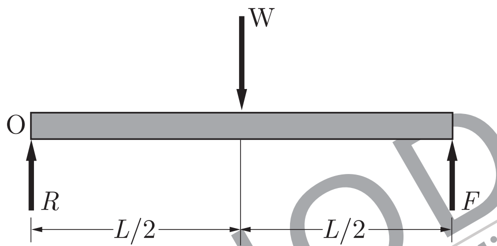
$$\sigma_1 = \frac{pr}{t}$$

and longitudinal or axial stress is

$$\sigma_2 = \frac{pr}{2t}$$

Ratio $$\frac{\sigma_1}{\sigma_2} = \frac{pr}{t} \times \frac{t}{pr} = 2$$

SOL 2.82 Option (B) is correct.



When the Force F is suddenly remove, then due to W, the rod is in rotating condition with angular acceleration α.

Thus equation of motion

$$\Sigma M_O = I_O \alpha$$

$$\frac{WL}{2} = I_O \alpha = \frac{mL}{}\alpha$$

or $$\frac{mgL}{2} = \frac{mL^2}{3}\alpha$$

or $$\alpha = \frac{3g}{2L}$$

Also the centre of the rod accelerate with linear acceleration a. Thus from *FBD* of rod

$$W - R = ma$$

$$mg - R = ma \quad \text{...(i)}$$

From the relation of linear and angular acceleration, we have

$$a = r\alpha$$

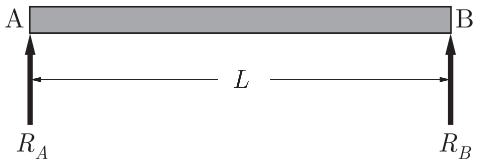
$$= \frac{L}{2} \times \frac{3g}{2L} = \frac{3g}{4}$$

Substitute this value in equation (i), we obtain

$$R = mg - m \times \frac{g}{}$$

$$R = \frac{mg}{4} = \frac{W}{4}$$

SOL 2.82 Option (B) is correct.



Total load on the beam

$$F = \int_0^L \sin\left(\frac{3\pi x}{L}\right) dx$$
$$= \left[-\cos\left(\frac{3\pi x}{L}\right) \times \frac{L}{3\pi}\right]_0^L$$
$$= -\left[\frac{-L}{3\pi} - \frac{L}{3\pi}\right] = \frac{2L}{3\pi}$$

This load acting at the centre of the beam because of the sin function. Now taking the moment about point B, we have

$$\Sigma M_B = 0$$
$$R_A \times L = \frac{2L}{3\pi} \times \frac{L}{2}$$
$$R_A = \frac{L}{3\pi}$$

SOL 2.82 Option (D) is correct.

Given $F_{\min} =$ kN, $F_{ma} =$ kN, $\sigma_y =$ a = N mm , $FOS =$ $\sigma_e =$ a = N mm

$$\sigma_{\min} = \frac{F_{\min}}{\text{Area}} = \frac{20 \times 10^3}{A}$$
$$\sigma_{\max} = \frac{F_{\max}}{\text{Area}} = \frac{100 \times 10^3}{A}$$

Now
$$\sigma_{\text{mean}} = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$
$$= \frac{120 \times 10^3}{2A} = \frac{60 \times 10^3}{A}$$

and
$$\sigma_v = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$
$$= \frac{80 \times 10^3}{2A} = \frac{40 \times 10^3}{A}$$

According to soderberg's criterion

$$\frac{}{FOS} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

or
$$\frac{1}{2} = \frac{60 \times 10^3}{240A} + \frac{40 \times 10^3}{160A}$$

or
$$\frac{1}{2} = \frac{10^3}{4A} + \frac{10^3}{4A}$$

or
$$A = 1000\,\text{mm}^2$$

SOL 2.82 Option (D) is correct.

For thin walled spherical shell circumferential (hoop) stress is

$$\sigma = \frac{pd}{4t} = \frac{pr}{2t}$$

For initial condition let radius r and thickness t, then

$$\sigma_1 = \frac{pr_1}{2t_1} \qquad ...(i)$$

For final condition radius r increased by 1%, then

$$r = r_1 + \frac{r_1}{1} = 1.\ 1 r_1$$

Thickness t decreased by 1% then

$$t = t_1 - \frac{t_1}{1} = \quad t_1$$

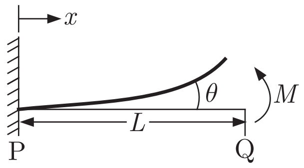
and $$\sigma_2 = \frac{pr_2}{2t_2} = \frac{p \times 1.01 r_1}{1 \times 9.99 t_1} = 1.0202 \frac{pr_1}{2t_1}$$

From Eq. (i) $$\sigma_2 = 1.0202 \times \sigma_1$$

Change in hoop stress (%)

$$\sigma_c = \frac{\sigma_2 - \sigma_1}{\sigma_1} \times 100 = \frac{1.0202\sigma_1 - \sigma_1}{\sigma_1} \times 100 = 2.02\%$$

SOL 2.82 Option (A) is correct.



Since $$EI\frac{d\ \ y}{dx} = M$$

Integrating $$EI\frac{dy}{dx} = mx + C_1 \quad ...(i)$$

At $x =$ $$\frac{dy}{dx} = 0$$

So $$EI(\ ) = M(\ ) + C_1 \Rightarrow C_1 = 0$$

Hence Eq.(i) becomes

$$EI\frac{dy}{dx} = mx$$

Again integrating $$EIy = \frac{mx^2}{2} + C_2 \quad ...(ii)$$

At $x =$ $y =$, $$EI(\ ) = \frac{m(0)^2}{2} + C_2$$

$$C = 0$$

Then Eq. (ii) becomes

$$EIy = \frac{Mx^2}{2}$$

$$y = \frac{Mx^2}{2EI} \qquad \Rightarrow y_{max} = \frac{ML}{EI}\ (\text{At } x = L, y = y_{max})$$

SOL 2.82 Option (C) is correct.

Critical buckling load, $$= \frac{\pi\ EI}{L} \quad ...(i)$$

For both ends clamped $L = \frac{L}{2}$

For both ends hinged $L = L$

Ratio for both ends clamped to both ends hinged is $= \frac{\frac{\pi\ EI}{(\frac{L}{\ })}}{\frac{\pi\ EI}{L}} = \frac{\ }{L} \times \frac{L}{1} =$

SOL 2.82 Option (B) is correct.

According to Von Mises Yield criterion

$$\sigma_Y = \frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$$

Given, $$T = \begin{bmatrix} 10 & 5 & 0 \\ 5 & 20 & 0 \\ 0 & 0 & -10 \end{bmatrix}$$

From given Matrix

$$\sigma_x = 10 \qquad \tau_{xy} = 5$$
$$\sigma_y = 20 \qquad \tau_{yz} = 0$$
$$\sigma_z = -10 \qquad \tau_{zx} = 0$$

So, $$\sigma_Y = \frac{1}{2}[(10-20)^2 + (20+10)^2 + (-10-10)^2 + 6(5^2+0^2+0^2)]$$
$$= \frac{1}{2} \times [100 + 900 + 400 + (6 \times 25)] = 27.83\text{ MPa}$$

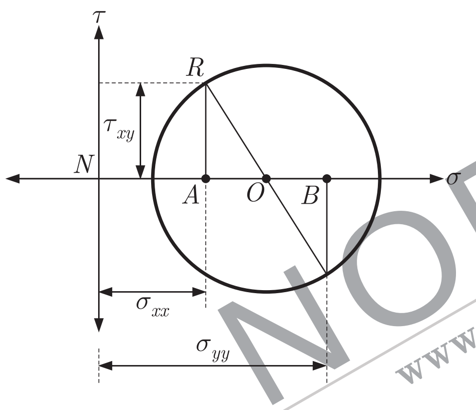
Shear yield stress

$$\tau_Y = \frac{\sigma_Y}{\sqrt{3}} = \frac{27.83}{\sqrt{3}} = 16.06\text{ MPa}$$

SOL 2.82 Option (B) is correct.

Given, $\sigma_{xx} = 40\text{ MPa} = AN$, $\sigma_{yy} = 100\text{ MPa} = BN$, $\tau_{xy} = 40\text{ MPa} = AR$

Diagram for Mohr's circle



Radius of Mohr's circle $$OR = \sqrt{(AR)^2 + (AO)^2}$$
$$AO = \frac{AB}{} = \frac{BN - AN}{2} = \frac{100-40}{2} = 30$$

Therefore, $$OR = \sqrt{(40)^2 + (30)^2} = 50\text{ MPa}$$

SOL 2.82 Option (A) is correct.

For a solid cube strain in x, y and z axis are

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\upsilon(\sigma_y + \sigma_z)}{E}$$
$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\upsilon(\sigma_x + \sigma_z)}{E}$$
$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\upsilon(\sigma_x + \sigma_y)}{E}$$

From symmetry of cube, $\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon$

and $\sigma_x = \sigma_y = \sigma_z = \sigma$

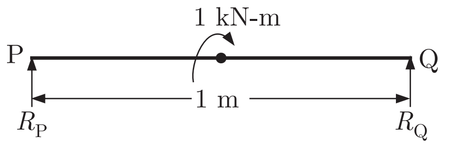
So $$\varepsilon = \frac{(\quad - \quad \upsilon)}{E} \times \sigma$$

Where $\varepsilon = -\alpha \Delta T$ (Thermal compression stress)

Therefore, $$\sigma = \frac{\varepsilon \times E}{(1-2\upsilon)} = -\frac{-\alpha \Delta T E}{(1-2\upsilon)} = -\frac{\alpha \Delta T E}{(1-2\upsilon)}$$

SOL 2.82 Option (A) is correct.

First of all we have to make a free body diagram of the given beam.



Here R_P and R_Q are the reaction forces acting at P and Q.
For equilibrium of forces on the beam,

$$R_P + R_Q = 0 \qquad ...(i)$$

Taking the moment about the point P,

$$R_Q \times \quad = 1\,\text{kN-m} \qquad \Rightarrow \quad R_Q = 1\,\text{kN-m}$$

From equation (i), $\quad R_P = - R_Q = -1\,\text{kN-m}$

Since, our assumption that R_P acting in the upward direction, is wrong,
So, R_P acting in downward direction and R_Q acting in upward direction.

SOL 2.82 Option (B) is correct.
Given : $l = 1\,\text{meter},\ b = 20\,\text{mm},\ h = 10\,\text{mm}$

We know that, $\quad$ Slenderness ratio $= \dfrac{l}{k}$

Where, $$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{bh}{b \times h}}$$

Substitute the values, we get

$$k = \sqrt{\frac{\frac{1}{12} \times 20 \times (10)^3 \times 10^{-12}}{10 \times 20 \times 10^{-6}}} = \sqrt{\frac{20 \times 10^{-3}}{12 \times 10 \times 20}}$$

$$= \sqrt{8.33 \times 10^{-6}} = 2.88 \times 10^{-3}\,\text{m}$$

$$\text{Slenderness ratio} = \frac{1}{2.88 \times 10^{-3}} = 347.22 \simeq 346$$

SOL 2.82 Option (C) is correct.
(P) Maximum-normal stress criterion → (M)
(Q) Maximum-distortion energy criterion → (N)
(R) Maximum-shear-stress criterion → (L)
So correct pairs are, P-M, Q-N, R-L

SOL 2.82 Option (B) is correct.
Given : $r = 500\,\text{mm},\ t = 10\,\text{mm},\ p = 5\,\text{MPa}$
We know that average circumferential (hoop) stress is given by,

$$\sigma_h = \frac{pd}{2t} = \frac{5 \times (2 \times 500)}{2 \times 10} = 250\,\text{MPa}$$

SOL 2.82 Option (B) is correct.
Here we see that shafts are in series combination. For series combination Total angular twist,

$$\theta = \theta_1 + \theta_2 \qquad ...(i)$$

From the torsional equation,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \qquad \Rightarrow \quad \theta = \frac{Tl}{GJ} \qquad J = \frac{\pi}{32} d^4$$

$$\theta = \frac{Tl}{\pi d \ G}$$

Now, from equation (i),

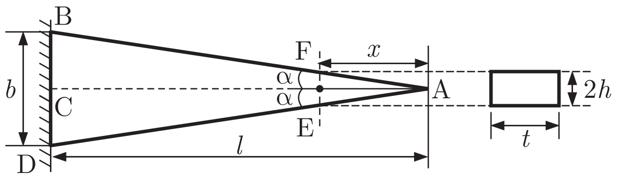
$$\theta = \frac{32\,T(L)}{\pi(2d)^4 G} + \frac{32\,T\left(\frac{L}{2}\right)}{\pi d^4 G} = \frac{TL}{\pi d\ \ G}\left[- + -\right] = \frac{TL}{\pi d\ \ G} \times - = \frac{TL}{\pi d\ \ G}$$

$$d = \left(\frac{TL}{\pi\theta G}\right)^{-}$$

SOL 2.82 Option (B) is correct.

Let, b = width of the base of triangle $ABD = BD$

t = thickness of conilever beam



From the similar triangle (Figure (i)) ΔABC or ΔAFE

$$\frac{b}{l} = \frac{h}{x} \qquad \text{let } OE = h$$

$$h = \frac{bx}{2l} \qquad \text{...(i)}$$

Now from figure (ii), For a rectangular cross section,

$$I = \frac{(2h)\,t^3}{12} = 2 \times \frac{bx}{2l} \times \frac{t^3}{12} = \frac{bxt^3}{12l} \qquad \text{From equation (i)}$$

SOL 2.82 Option (D) is correct.

We know that deflection equation is

$$EI\frac{d\ \ t}{dx} = M = P \times x$$

$$\frac{d\ \ y}{dx} = \frac{}{EI}P \times x$$

From previous part of the question

$$\frac{d\ \ y}{dx} = \frac{}{E \times \frac{bxt}{L}} \times Px = \frac{PL}{Ebt}$$

On Integrating, we get

$$\frac{dy}{dx} = \frac{PLx}{Ebt} + C \qquad \text{...(i)}$$

When $x = L$, $\frac{dy}{dx} =$

So, $$0 = \frac{PL}{Ebt} + C \quad \Rightarrow \quad C = -\frac{PL}{Ebt}$$

Again integrating equation (i),

$$y = \frac{PL}{Ebt} \times \frac{x}{} + C\ x + C \qquad \text{...(ii)}$$

When $x = L$, $y =$

So, $$0 = \frac{12PL}{2Ebt^3} \times L^2 + C_1 L + C_2 = \frac{PL}{Ebt} - \frac{PL}{Ebt} + C$$

$$C = \frac{PL}{Ebt}$$

From equation (ii),

$$y = \frac{PLx}{Ebt} - \frac{PL\ x}{Ebt} + \frac{PL}{Ebt} \qquad \text{...(iii)}$$

The maximum deflection occurs at $x =$, from equation (iii),

$$y_{max} = 0 + 0 + \frac{6PL^3}{Ebt^3} = \frac{PL}{Ebt}$$

SOL 2.82 Option (C) is correct.

Given : $\sigma_x = -200\,\text{MPa}$, $\sigma_y = 100\,\text{MPa}$, $\tau_{xy} = 100\,\text{MPa}$

We know that maximum shear stress is given by,

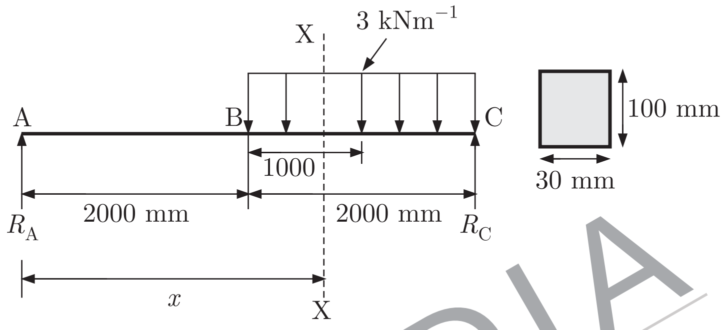
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Substitute the values, we get

$$\tau_{max} = \frac{1}{2}\sqrt{(-200 - 100)^2 + 4 \times (100)^2}$$

$$= \frac{1}{2}\sqrt{90000 + 40000} = 180.27 \simeq 180.3\,\text{MPa}$$

SOL 2.82 Option (C) is correct.



First of all we have to make the FBD of the given system.

Let R_A and R_C are the reactions acting at point A and C respectively.

In the equilibrium condition of forces,

$$R_A + R_C = 6000\,\text{N} \qquad \text{...(i)}$$

Taking moment about point A,

$$R_C \times \quad = 6000 \times 3$$

$$R_C = \frac{18000}{4} = 4500\,\text{N} = 4.5\,\text{kN}$$

And from equation (i),

$$R_A = 6000 - 4500 = 1500\,\text{N} = 1.5\,\text{kN}$$

Taking a section $X - X$ at a distance x from A and taking the moment about this section

$$M_{XX} = R_A \times x - \quad (x - \quad) \times \frac{(x - \quad)}{} \qquad F = \quad (x - \quad) \text{ and } d = \frac{x -}{}$$

$$= 1.5x - 1.5(x - 2)^2 \qquad \text{...(ii)}$$

For maximum Bending moment,

$$\frac{d}{dx}(M_{XX}) = 0$$

$$1.5 - 2 \times 1.5(x - 2) = 0$$

$$1.5 - 3x + 6 = 0$$

$$-3x = -7.5$$

$$x = 2.5\,\text{m} = 2500\,\text{mm}$$

So the maximum bending moment occurs at 2500 mm to the right of A.

SOL 2.82 Option (B) is correct.

From the equation (ii) of the previous part, we have

Maximum bending moment at $x =$ m is,

$$(BM)_{2.5\,\text{m}} = 1.5 \times 2.5 - 1.5(2.5-2)^2 = 3.375\ \text{kN-m}$$

From the bending equation,

$$\sigma_b = \frac{M}{I} \times y = \frac{M}{\frac{bh^3}{12}} \times \frac{h}{2} = \frac{M}{bh}$$

Substitute the values, we get

$$\sigma_b = \frac{6 \times 3375}{0.030 \times (0.1)^2} = 67.5 \times 10^6\ \text{N/m}^2 = 67.5\ \text{MPa}$$

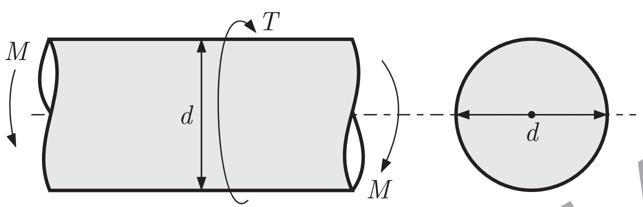
SOL 2.82 Option (C) is correct.

Given : $\sigma_1 = 100\ \text{MPa}$, $\sigma_2 = 40\ \text{MPa}$

We know, the maximum shear stress for the plane complex stress is given by

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100-40}{2} = \frac{60}{2} = 30\ \text{MPa}$$

SOL 2.82 Option (C) is correct.



We know that, for a shaft of diameter d is subjected to combined bending moment M and torque T, the equivalent Torque is,

$$T_e = \sqrt{M + T}$$

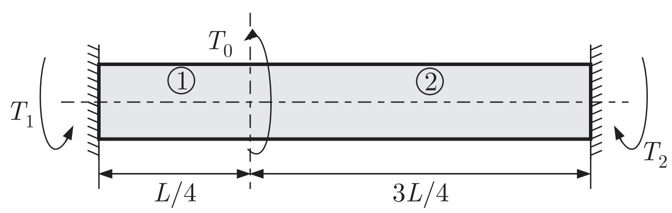
Induced shear stress is,

$$\tau = \frac{T}{\pi d} = \frac{}{\pi d} \times \sqrt{M + T}$$

Now, for safe design, τ should be less than $\frac{S_{sy}}{N}$

Where, $S_{sy} =$ Torsional yield strength and $N =$ Factor of safety

SOL 2.82 Option (B) is correct.



First, the shaft is divided in two parts (1) and (2) and gives a twisting moment T_1 (in counter-clockwise direction) and T_2 (in clock wise direction) respectively. By the nature of these twisting moments, we can say that shafts are in parallel combination.

So, $T = T_1 + T_2$ ...(i)

From the torsional equation,

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \quad \Rightarrow \quad T = \frac{GJ\theta}{l}$$

But, here $G_1 = G_2$

$\theta_1 = \theta_2$ For parallel connection

$$J = J \qquad \text{Diameter is same}$$

So, $$T\,l = T\,l$$

$$T \times \frac{L}{} = T \times \frac{L}{}$$

$$T = 3T_2$$

Now, From equation (i),

$$T = 3T_2 + T_2 = 4T_2$$

$$T = \frac{T_0}{4}$$

And $$T = \frac{3T_0}{4}$$

Here $$T > T$$

So, maximum shear stress is developed due to T,

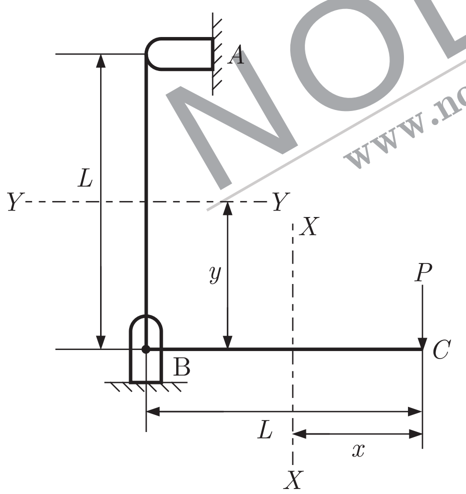
$$\frac{T}{J} = \frac{\tau_{\max}}{r} \qquad \Rightarrow \quad \tau_{\max} = \frac{T}{J} \times r$$

Substitute the values, we get

$$\tau_{\max} = \frac{\left(\frac{3T_0}{4}\right)}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{32 \times 3T_0}{8\pi \times d^3} = \frac{T}{\pi d}$$

SOL 2.82 Option (D) is correct.

We have to solve this by Castigliano's theorem.



We have to take sections XX and YY along the arm BC and AB respectively and find the total strain energy.

So, Strain energy in arm BC is,

$$U_{BC} = \int_0^L \frac{M_x^2}{2EI}dx = \int_0^L \frac{(Px)^2}{2EI}dx \qquad M_x = P \times x$$

Integrating the equation and putting the limits, we get

$$U_{BC} = \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_0^L = \frac{P^2L^3}{6EI}$$

Similarly for arm AB, we have

$$U_{AB} = \int_0^L \frac{M_y^2}{2EI}dy = \int_0^L \frac{P^2L^2}{2EI}dy \qquad M_y = P \times L$$

$$= \frac{P^2L^3}{2EI}$$

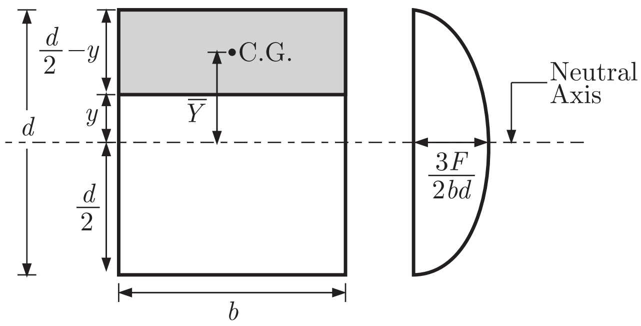
So, total strain energy stored in both the arms is,

$$U = U_{AB} + U_{BC} = \frac{P^2L^3}{2EI} + \frac{P^2L^3}{6EI} = \frac{2P^2L^3}{3EI}$$

From the Castigliano's theorem, vertical deflection at point A is,

$$\delta_A = \frac{\delta U}{\delta P} = \frac{\delta}{\delta P}\left(\frac{P\ L}{EI}\right) = \frac{PL}{EI}$$

SOL 2.82 Option (D) is correct.



For a rectangle cross-section:

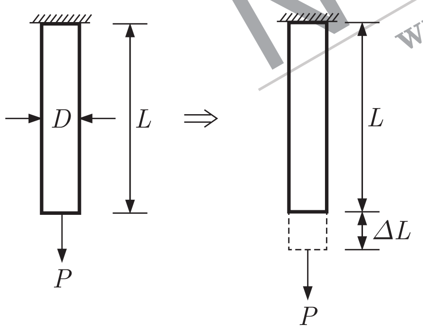
$$\tau_v = \frac{FA\overline{Y}}{Ib} = \frac{F}{bd}\left(\frac{d}{} - y\right) \qquad F = \text{Transverse shear load}$$

Maximum values of τ_v occurs at the neutral axis where, $y =$

$$\text{Maximum } \tau_v = \frac{F}{bd} \times \frac{d}{} = \frac{F}{bd} = \frac{3}{2}\tau_{\text{mean}} \qquad \tau_{mean} = \frac{F}{bd}$$

So, transverse shear stress is variable with maximum on the neutral axis.

SOL 2.82 Option (D) is correct.



From the application of load P, the length of the rod increases by an amount of ΔL

$$\Delta L = \frac{PL}{AE} = \frac{PL}{\frac{\pi}{4}D^2E} = \frac{4PL}{\pi D^2 E}$$

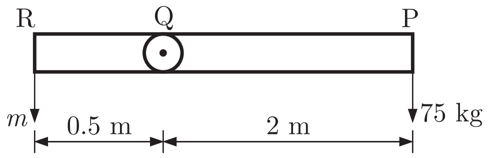
And increase in length due to applied load P in axial or longitudinal direction, the shear modulus is comes in action.

$$G = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{\tau_s}{\Delta L/L} = \frac{\tau_s L}{\Delta L}$$

So, for calculating the resulting change in diameter both young's modulus and shear modulus are used.

SOL 2.82 Option (D) is correct.

First of all we have to make the FBD of the given system.

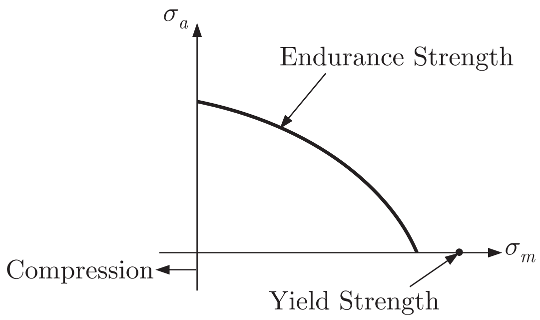


Let mass of the counter weight $= m$.
Here point Q is the point of contraflexure or point of inflection or a virtual hinge.

So, $$M_Q = 0$$

$$m \times \; . \; = 75 \times 2 \qquad \Rightarrow \quad m = 300 \text{ kg}$$

SOL 2.82 Option (D) is correct.



(Gerber's Parabola)

The figure shown the Gerber's parabola. It is the characteristic curve of the fatigue life of the shaft in the presence of the residual compressive stress.
The fatigue life of the material is effectively increased by the introduction of a compressive mean stress, whether applied or residual.

SOL 2.82 Option (D) is correct.
Here corner point P is fixed. At point P double stresses are acting, one is due to bending and other stress is due to the direct Load.
So, bending stress, (From the bending equation)

$$\sigma_b = \frac{M}{I} y$$

Distance from the neutral axis to the external fibre $y = \frac{b}{\;} = b$,

$$\sigma_b = \frac{F(L-b)}{\frac{(2b)^4}{12}} \times b \qquad \text{For square section } I = \frac{b}{\;}$$

$$= \frac{12F(L-b)}{16b^3} = \frac{3F(L-b)}{4b^3}$$

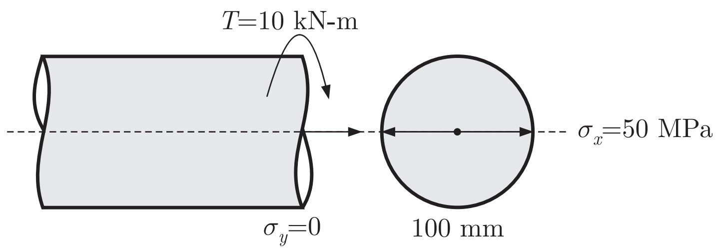
and direct stress,

$$\sigma_d = \frac{F}{(2b)^2} = \frac{F}{4b^2} = \frac{F}{4b^2} \times \frac{b}{b} = \frac{Fb}{4b^3}$$

Total axial stress at the corner point P is,

$$\sigma = \sigma_b + \sigma_d = \frac{3F(L-b)}{4b^3} + \frac{Fb}{4b^3} = \frac{F(3L-2b)}{4b^3}$$

SOL 2.82 Option (B) is correct.



The shaft is subjected to a torque of 10 kN-m and due to this shear stress is developed in the shaft,

$$\tau_{xy} = \frac{T}{J} \times r = \frac{10 \times 10^3}{\frac{\pi}{32} d^4} \times \frac{d}{2} \qquad \text{From Torsional equation}$$

$$= \frac{\times \quad \times}{\pi d} = \frac{16 \times 10^4}{3.14 \times (10^{-1})^3} = \frac{160}{3.14} = 50.95\,\text{MPa}$$

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Substitute the values, we get

$$\sigma_1 = \frac{50}{2} + \frac{1}{2}\sqrt{(50)^2 + 4 \times (50.95)^2} = 25 + \frac{1}{2}\sqrt{12883.61}$$

$$= 25 + \frac{113.50}{2} = 25 + 56.75 = 81.75\,\text{MPa} \simeq 82\,\text{MPa}$$

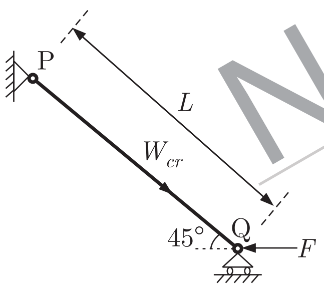
SOL 2.82 Option (B) is correct.

We know that according to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is represented by the general equation,

$$W_{cr} = \frac{C\pi \; EI}{L} \qquad \text{...(i)}$$

Where L = length of column

C = Constant, representing the end conditions of the column.



Here both ends are hinged, $C = 1$

From equation (i), $W_{cr} = \frac{\pi \; EI}{L}$

Minimum force F required , $W_{cr} = F\cos \quad ^\circ$

$$F = \frac{W_{cr}}{\cos 45^\circ} = \frac{\sqrt{2}\,\pi^2 EI}{L^2}$$

SOL 2.82 Option (D) is correct.

We know that deflection in a compression spring is given by

$$\delta = \frac{PR \;\; n}{d \;\; G} = \frac{PD \;\; n}{d \;\; G}$$

Where n = number of active coils

D = Mean coil Diameter

d = Music wire Diameter

And $k = \frac{P}{\delta} = \frac{d \;\; G}{D \;\; n}$

$$k \propto \frac{\quad}{D}$$

Given that mean coil diameter is reduced to 10 mm.

So, $D_1 = 20\,\text{mm}$

$D = 20 - 10 = 10\,\text{mm}$

and $\frac{k}{k_1} = \left(\frac{D_1}{D}\right) = \left(\frac{0}{10}\right) =$

$k = 8k_1$

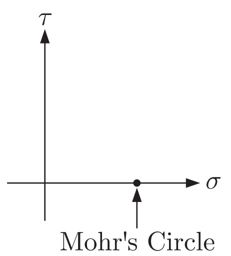
So, stiffness is increased by 8 times.

SOL 2.82 Option (B) is correct.

Pressure will remain uniform in all directions. So, hydrostatic load acts in all directions on the fluid element and Mohr's circle becomes a point on $\sigma - \tau$ axis and

$\sigma_x = \sigma_y$ and $\tau_{xy} = 0$

So, $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = 0$$



SOL 2.82 Option (B) is correct.

Given : $R = 1\,\text{m}$, $t = 1\,\text{mm} = 10^{-3}\,\text{m}$

We know that axial or longitudinal stress for a thin cylinder is,

$$\sigma_x = \sigma_a = \frac{p \times D}{4t} = \frac{p \times 2R}{4t} \qquad \text{...(i)}$$

Here, p = Pressure of the fluid inside the shell

So, pressure at 1 m depth is,

$$p = \rho g h = 1000 \times 10 \times 1 = 10^4\,\text{N/m}^2$$

From equation (i),

$$\sigma_a = \frac{10^4 \times 2 \times 1}{4 \times 10^{-3}} = 5 \times 10^6\,\text{N/m}^2 = 5\,\text{MPa}$$

and hoop or circumferential stress,

$$\sigma_y = \sigma_c = \frac{p \times D}{2t} = \frac{10^4 \times 2}{2 \times 10^{-3}} = 10 \times 10^6\,\text{N/m}^2 = 10\,\text{MPa}$$

SOL 2.82 Option (A) is correct.

Given : υ or $\frac{1}{m} = 0.3$, $E = 100\,\text{GPa} = 100 \times 10^9\,\text{Pa}$

Axial strain or longitudinal strain at mid – depth is,

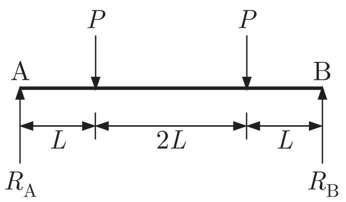
$$\sigma_a = \sigma_x = \frac{pD}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

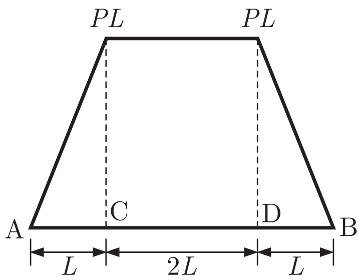
Substitute the values, we get

$$\sigma_a = \frac{10^4 \times 2 \times 1}{2 \times 10^{-3} \times 100 \times 10^9}\left(\frac{1}{2} - 0.3\right)$$

$$= \frac{10^4}{10^8}\left(\frac{1}{2} - 0.3\right) = 10^{-4} \times 0.2 = 2 \times 10^{-5}$$

SOL 2.82 Option (C) is correct.





B.M.D.

In equilibrium condition of forces,

$$R_A + R_B = 2P \qquad ...(i)$$

Taking the moment about point A,

$$R_B \times \quad L - P \times L - P \times \quad L = 0$$

$$R_B \times \quad L - \quad PL = 0$$

$$R_B = \frac{4PL}{4L} = P$$

From equation (i), $R_A = 2P - P = P$

With the help of R_A and R_B, we have to make the Bending moment diagram of the given beam. From this B.M.D, at section AC and BD Bending moment varying with distance but at section CD, it is constant.

Now strain energy $U = \int \frac{M^2}{2EI} dx$

Where M is the bending moment of beam.

Total strain energy is given by

$$U = \underbrace{\int_0^L \frac{(Px)^2 dx}{2EI}}_{\{\text{for section } AC\}} + \underbrace{\frac{(PL)^2 2L}{2EI}}_{\{\text{for section } CD\}} + \underbrace{\int_0^L \frac{(Px)^2 dx}{2EI}}_{\{\text{for section } BD\}}$$

$$= 2\int_0^L \frac{(Px)^2 dx}{2EI} + \frac{P^2 L^3}{EI} = \frac{P}{EI}\int^L x \; dx + \frac{P \quad L}{EI}$$

Integrating above equation, we get

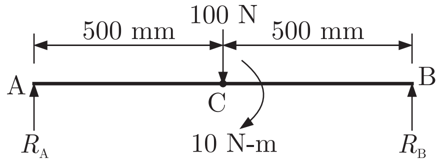
$$U = \frac{P}{EI}\left[\frac{x}{\quad}\right]^L + \frac{P \quad L}{EI} = \frac{P^2 L^3}{3EI} + \frac{P^2 L^3}{EI} = \frac{4P^2 L^3}{3EI}$$

SOL 2.82 Option (B) is correct.

Due to 100 N force, bending moment occurs at point C and magnitude of this bending moment is,

$$M_C = 100 \times (0.1) = 10 \text{ N–m} \qquad \text{(in clock wise direction)}$$

We have to make a free body diagram of the given beam,



Where R_A and R_B are the reactions acting at point A and B

For equilibrium of forces,

$$R_A + R_B = 100\,\text{N} \qquad \text{...(i)}$$

Taking the moment about point A,

$$100 \times 0.5 + 10 = R_B \times \qquad \Rightarrow R_B = \quad \text{N}$$

From equation (i),

$$R_A = 100 - R_B = 100 - 60 = 40\,\text{N}$$

Maximum bending moment occurs at point C,

$$M_C = R_A \times \; . \; + \quad = 40 \times 0.5 + 10 = 20 + 10 = 30\,\text{N–m}$$

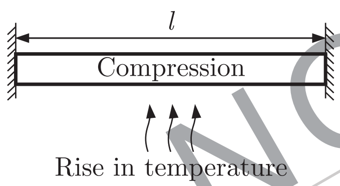
SOL 2.82 Option (C) is correct.

Let, l = original length of the bar

α = Co-efficient of linear expansion of the bar material

ΔT = Rise or drop in temperature of the bar

δl = Change in length which would have occurred due to difference of temperature if the ends of the bar were free to expand or contract.



$$\alpha = \frac{\delta l}{l \times \Delta T}$$

or, $$\delta l = l \times \alpha \times \Delta T$$

And temperature strain,

$$\varepsilon = \frac{\delta l}{l} = \frac{l \times \alpha \times \Delta T}{l} = \alpha \times \Delta T$$

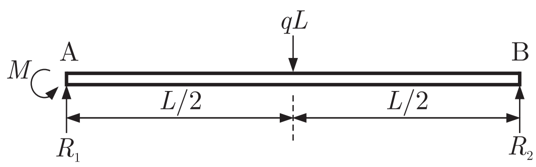
Basically temperature stress and strain are longitudinal (i.e. tensile or compressive) stress and strain

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$$

$$\sigma = E\varepsilon = E\alpha \Delta T$$

SOL 2.82 Option (A) is correct.

First of all, we have to make a FBD of the beam. We know that a UDL acting at the mid-point of the beam and its magnitude is equal to $(q \times L)$. So,



In equilibrium of forces,

$$R + R = qL \qquad \text{...(i)}$$

This cantilever beam is subjected to two types of load.

First load is due to UDL and second load is due to point load at B. Due to this deflection occurs at B, which is equal in amount.

So, deflection occurs at B due to the UDL alone,

$$\delta_{UDL} = \frac{qL^4}{8EI}$$

Also, deflection at B due to point load,

$$\delta_{PL} = \frac{R_2 L^3}{3EI}$$

Deflections are equal at B,

$$\delta_{UDL} = \delta_{PL}$$

$$\frac{qL^4}{8EI} = \frac{R_2 L^3}{3EI} \qquad \Rightarrow \quad R = \frac{3qL}{8}$$

And from equation (i), we have

$$R = qL - R = qL - \frac{qL}{} = \frac{5qL}{8}$$

For M, taking the moment about B,

$$-qL \times \frac{L}{} + R \times L - M = 0$$

$$-\frac{qL^2}{2} + \frac{5qL^2}{8} - M = 0$$

$$M = \frac{qL^2}{8}$$

Therefore, $R = \frac{qL}{}$, $R = \frac{qL}{}$ and $M = \frac{qL}{}$

SOL 2.82 Option (B) is correct.

Given : $\nu = 200 \times 100 \times 50\,\text{mm}^3 = 10^6\,\text{mm}^3$

$$p = 15\,\text{MPa} = 15 \times 10^6\,\text{N/m}^2 = 15\,\text{N/mm}^2$$

$$E = 200\,\text{GPa} = 200 \times 10^3\,\text{N/mm}^2$$

$$\left(\upsilon \text{ or } \frac{1}{m}\right) = 0.3$$

We know the relation between volumetric strain, young's modulus and Poisson's ration is given by,

$$\frac{\Delta\nu}{\nu} = \frac{p}{E}(\;\; - \;\; \upsilon)$$

Substitute the values, we get

$$\frac{\Delta\nu}{10^6} = \frac{3 \times 15}{200 \times 10^3}(1 - 2 \times 0.3)$$

$$\Delta\nu = \frac{45 \times 10}{2}(1 - 0.6) = 225 \times 0.4 = 90\,\text{mm}^3$$

SOL 2.82 Option (C) is correct.

Given : $T =$ -m $=$ -mm, $G =$ GPa $= 80 \times 10^3\,\text{N/mm}^2$

$L = L =$ mm, $d = 50$ mm, $d = 25$ mm

We know that for a shaft of length l and polar moment of inertia J, subjected to a torque T with an angle of twist θ. The expression of strain energy,

$$U = \frac{1}{2}\frac{T^2 l}{GJ} \qquad\qquad U = -T\theta, \text{ and } \theta = \frac{Tl}{GJ}$$

So Total strain energy,

$$U = \frac{T^2L}{2GJ_1} + \frac{T^2L}{2GJ_2} = \frac{T^2L}{2G}\left[\frac{1}{J_1} + \frac{1}{J_2}\right] \qquad J = \frac{\pi}{}d$$

Substitute the values, we get

$$U = \frac{(10^4)^2 \times 100}{2 \times 80 \times 10^3}\left[\frac{1}{\frac{\pi}{32}(50)^4} + \frac{1}{\frac{\pi}{32}(25)^4}\right]$$

$$= \frac{10^6}{16} \times \frac{32}{\pi}\left[\frac{1}{625 \times 10^4} + \frac{1}{390625}\right]$$

$$= \frac{10^6}{16 \times 10^4} \times \frac{32}{\pi}\left[\frac{1}{625} + \frac{1}{39.0625}\right]$$

$$= 63.69 \times [0.0016 + 0.0256] = 1.73 \text{ N-mm}$$

SOL 2.82 Option (A) is correct.

Given : $F = 600$ N (Parallel to Z-direction), $d = 30$ mm

Normal stress at point P, from bending equation

$$\sigma = \frac{M}{I} \times y = \frac{600 \times 300}{\frac{\pi}{64}d^4} \times \frac{d}{2} \qquad \text{Here } M = \text{bending moment}$$

$$= \frac{\times \quad \times}{\pi d} = \frac{18 \times 10^4 \times 32}{3.14(30)^3} = 67.9 \text{ MPa}$$

And from Torsional equation, shear stress, $\frac{T}{J} = \frac{\tau}{r}$

$$\tau = \frac{T}{J} \times r = \frac{600 \times 500}{\frac{\pi}{32}d^4} \times \frac{d}{2} \qquad T = \text{Force} \times \text{Area length}$$

$$= \frac{16 \times 600 \times 500}{3.14 \times (30)^3} = 56.61 \text{ MPa}$$

SOL 2.82 Option (D) is correct.

Here : $\sigma_x = 0$, $\sigma_y = 67.9$ MPa, $\tau_{xy} = 56.6$ MPa

Maximum principal stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \sigma_x =$$

Substitute the values, we get

$$\sigma_1 = \frac{0 + 67.9}{2} + \frac{1}{2}\sqrt{(-67.9)^2 + 4 \times (56.6)^2}$$

$$= 33.95 + \frac{1}{2}\sqrt{17424.65} = 33.95 + 66$$

$$= 99.95 \simeq 100 \text{ MPa}$$

And $$\tan 2\theta = \frac{\tau_{xy}}{\sigma_x - \sigma_y}$$

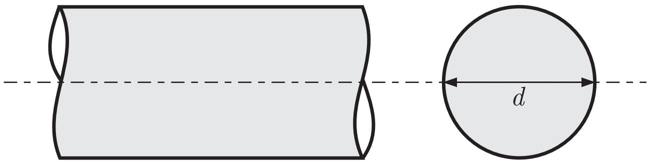
Substitute the values, we get

$$\tan 2\theta = \frac{2 \times 56.6}{0 - 67.9} = -1.667$$

$$2\theta = -59.04$$

$$\theta = -\frac{59.04}{2} = -29.52^\circ$$

SOL 2.82 Option (C) is correct.



From the Torsional equation

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$$

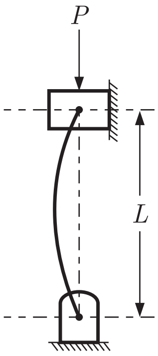
Take first two terms,

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\frac{T}{\frac{\pi}{32}d^4} = \frac{\tau}{\frac{d}{2}} \qquad J = \text{Polar moment of inertia}$$

$$\tau_{max} = \frac{T}{\pi d}$$

SOL 2.82 Option (D) is correct.



According to Euler's theory, the crippling or buckling load (P_{cr}) under various end conditions is represented by a general equation,

$$P_{cr} = \frac{C\pi\, EI}{L} \qquad ...(i)$$

Where, E = Modulus of elasticity

I = Mass-moment of inertia

L = Length of column

C = constant, representing the end conditions of the column or end fixity coefficient.

Here both ends are hinged, $C = 1$

Substitute in equation (i), we get $P_{cr} = \frac{\pi\, EI}{L}$

SOL 2.82 Option (C) is correct.

According to "VON MISES - HENKY THEORY", the elastic failure of a material occurs when the distortion energy of the material reaches the distortion energy at the elastic limit in simple tension.

Shear strain energy due to the principle stresses σ_1, σ_2 and σ_3

$$\Delta E = \frac{1+\upsilon}{6E}[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2]$$

$$= \frac{1+\upsilon}{6E}\left[2(\sigma_1^2+\sigma_2^2+\sigma_3^2) - 2(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$$

$$= \frac{1+v}{3E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]$$

SOL 2.82 Option (A) is correct.

Given : $A = (40)^2 = 1600\,\text{mm}^2$, $P = -200\,\text{kN}$ (Compressive)

$L = 2\,\text{m} = 2000\,\text{mm}$, $E = 200\,\text{GPa} = 200 \times 10^3\,\text{N/mm}^2$

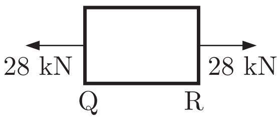
Elongation of the bar,

$$\Delta L = \frac{PL}{AE} = \frac{-200 \times 10^3 \times 2000}{1600 \times 200 \times 10^3} = -1.25\,\text{mm} \qquad \text{Compressive}$$

In magnitude, $\Delta L = 1.25\,\text{mm}$

SOL 2.82 Option (A) is correct.

The *FBD* of segment *QR* is shown below :



FBD

Given : $A = 700\,\text{mm}^2$

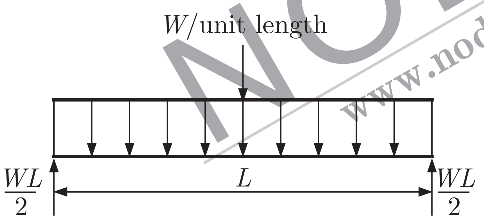
From the free body diagram of the segment *QR*,

Force acting on *QR*, $P = 28\,\text{kN}$ (Tensile)

Stress in segment *QR* is given by,

$$\sigma = \frac{P}{\text{Area}} = \frac{28 \times 10^3}{700 \times 10^{-6}} = 40\,\text{MPa}$$

SOL 2.82 Option none of these is correct.



Given : $L = 6\,\text{m}$, $W = 1.5\,\text{kN/m}$, $d = 75\,\text{mm}$

We know that for a uniformly distributed load, maximum bending moment at the centre is given by,

$$B.M. = \frac{WL^2}{8} = \frac{1.5 \times 10^3 \times (6)^2}{8}$$

$$B.M. = 6750\,\text{N–m} = 6.75\,\text{kN–m}$$

SOL 2.82 Option (A) is correct.

From the bending equation,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where M = Bending moment acting at the given section = 6.75 kN–m

I = Moment of inertia $= \frac{\pi}{64}d^4$

y = Distance from the neutral axis to the external fibre $= \frac{d}{2}$

σ_b = Bending stress

So, $$\sigma_b = \frac{M}{I} \times y$$

Substitute the values, we get

$$\sigma_b = \frac{6.75 \times 10^6}{\frac{\pi}{64}(75)^4} \times \frac{75}{2} = \frac{32400}{\pi \times 2 \times (75)^4} \times 10^6$$

$$= 1.6305 \times 10^{-4} \times 10^6 = 163.05\,\text{MPa} \simeq 162.98\,\text{MPa}$$

SOL 2.82 Option (A) is correct.

We know that due to temperature changes, dimensions of the material change. If these changes in the dimensions are prevented partially or fully, stresses are generated in the material and if the changes in the dimensions are not prevented, there will be no stress set up. (Zero stresses).

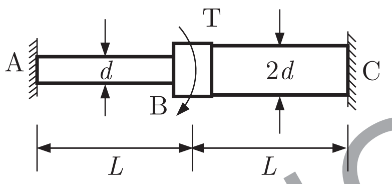
Hence cylindrical rod is allowed to expand or contract freely.

So, $\sigma_r =$ and $\sigma_z =$

SOL 2.82 Option (A) is correct.

From the figure, we can say that load P applies a force on upper cantilever and the reaction force also applied on upper cantilever by the rigid roller. Due to this, deflections are occur in both the cantilever, which are equal in amount. But because of different forces applied by the P and rigid roller, the slopes are unequal.

SOL 2.82 Option (C) is correct.



Here both the shafts AB and BC are in parallel connection.

So, deflection in both the shafts are equal.

$$\theta_{AB} = \theta_{BC} \quad \text{...(i)}$$

From Torsional formula,

$$\frac{T}{J} = \frac{G\theta}{L} \qquad \Rightarrow \theta = \frac{TL}{GJ}$$

From equation (i),

$$\frac{T_A L}{GJ_{AB}} = \frac{T_C L}{GJ_{BC}}$$

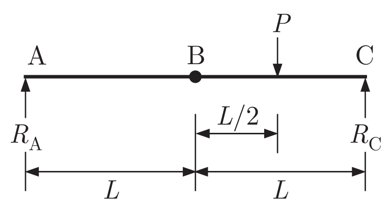
$$\frac{T_A \times L}{G \times \frac{\pi}{}d} = \frac{T_C \times L}{G \times \frac{\pi}{}(\ \ d)}$$

$$\frac{T_A}{d} = \frac{T_C}{16d^4} \qquad \text{For same material, } G_{AB} = G_{BC}$$

$$T_C = 16\,T_A$$

SOL 2.82 Option (B) is correct.

First of all we have to make a Free body diagram of the given beam.



Where, R_A and R_B are the reactions acting at point A and B

The point B is a point of contraflexure or point of inflexion or a virtual hinge. The characteristic of the point of contraflexure is that, about this point moment equal to zero.

For span BC, $\quad M_B = 0$

$$R_C \times L = P \times \frac{L}{}$$

$$R_C = \frac{P}{2}$$

For the equilibrium of forces on the beam,

$$R_A + R_C = P$$

$$R_A = P - \frac{P}{} = \frac{P}{2}$$

Now for the bending moment about point A, take the moment about point A,

$$M_A + R_C \times \quad L - P \times \left(L + \frac{L}{}\right) = 0$$

$$M_A + \frac{P}{} \times \quad L - P \times \frac{L}{} = 0$$

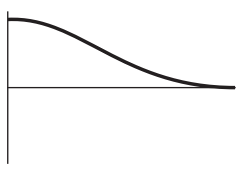
$$M_A = \frac{PL}{2}$$

SOL 2.82 Option (C) is correct.

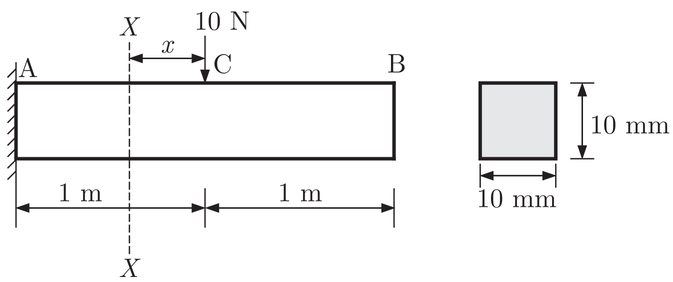
We know that, for a uniformly varying load bending moment will be cubic in nature.

(A) We see that there is no shear force at B, so the slope of BMD at right of B must be zero and similarly on left end A there is no shear force, so slope of BMD also zero.

(B) Now due to triangular shape of load intensity, when we move from right to left, the rate of increase of shear force decreases and maximum at the middle and therefore it reduces.



SOL 2.82 Option (A) is correct.



Taking a section XX on the beam.

Moment about this section XX

$$M_{XX} = 10 \times x = 10x \text{ N–m}$$

For a square section,

$$I = \frac{b^4}{12} = \frac{(10 \times 10^{-3})^4}{12} = \frac{10^{-8}}{12} \text{ m}^4$$

Using the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y} \qquad \Rightarrow \sigma = \frac{M}{I} y$$

Substitute the values, we get

$$\sigma = \frac{10x}{\frac{10^{-8}}{12}} \times \frac{10^{-2}}{2} = 60 \times 10^{6} x \qquad \text{...(i)}$$

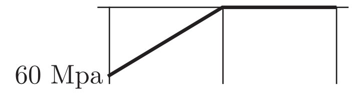
From equation (i), Bending stress at point $A(x = \ )$,

$$\sigma_A = 60 \times 10^6 \times 0 = 0$$

And at point C $(x = 1\,\text{m})$

$$\sigma_C = 60 \times 10^6 \times 1 = 60\,\text{MPa}$$

As no any forces are acting to the right of the point C.
So bending stress is constant after point C.



SOL 2.82 Option (C) is correct.

Maximum shear stress, $\tau = \frac{\sigma_{max} - \sigma_{min}}{2}$

Maximum shear stress at the elastic limit in simple tension (yield strength) $= \frac{\sigma_Y}{2}$

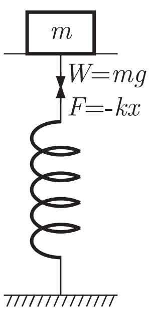
To prevent failure $\frac{\sigma_{max} - \sigma_{min}}{2} \le \frac{\sigma_Y}{2}$

$$\sigma_{max} - \sigma_{min} = \sigma_Y$$

Here $\sigma_{max} = -10\,\text{MPa}, \ \sigma_{min} = -100\,\text{MPa}$

So, $\sigma_Y = -10 - (-100) = 90\,\text{MPa}$

SOL 2.82 Option (B) is correct.
Initial length (un-deformed) of the spring $= L$ and spring stiffness $= k$



Let spring is deformed by an amount Δx, then Spring force, $F = k\Delta x$

For initial condition, $2g = k(L - \ . \ )$ $\qquad W = mg$ ...(i)

After this a mass of 20 kg is placed on the 2 kg pan. So total mass becomes 22 kg and length becomes 100 mm.

For this condition, $(20 + 2) g = k(L - 0.1)$ ...(ii)

Dividing equation (ii) by equation (i),

$$\frac{22g}{2g} = \frac{k(L - 0.1)}{k(L - 0. \ )}$$

$$11 = \frac{(L - 0.1)}{(L - 0.2)}$$

$$11L - 2.2 = L - 0.1$$

$$10L = 2.1$$

$$L = \frac{2.1}{10} = 0.21\,\text{m} = 210\,\text{mm}$$

And from equation (i),

$$2g = k(\quad - \quad)$$

$$k = \frac{2 \times 9.8}{0.01} = 1960\,\text{N/m}$$

So, $L = 210\,\text{mm}$, and $k = 1960\,\text{N/m}$

SOL 2.82 Option (A) is correct.

Relation between E, G and υ is given by,

$$E = 2G(1+\upsilon)$$

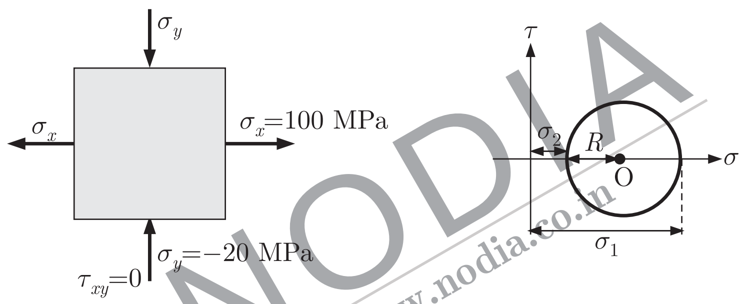
Where E = young's modulus

G = Shear Modulus

υ = Poisson's ratio

Now, $$\frac{E}{G} = 2(1+\upsilon)$$

SOL 2.82 Option (C) is correct.



$\sigma_x =$ MPa (Tensile), $\sigma_y = -$ MPa (Compressive)

We know that, $$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From the figure, Radius of Mohr's circle,

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2} \times 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Substitute the values, we get

$$R = \sqrt{\left[\frac{100 - (-20)}{2}\right]^2} = 60$$

SOL 2.82 Option (B) is correct.

Given : $T = 10$ N-m, $k_{MN} = 20$ N-m/rad, $k_{NO} = 30$ N-m/rad, $k_{OP} = 60$ N-m/rad

Angular deflection, $$\theta = \frac{T}{k}$$

For section MN, NO or OP, $\theta_{MN} = \frac{10}{20}$ rad, $\theta_{NO} = \frac{10}{30}$ rad, $\theta_{OP} = \frac{10}{60}$ rad

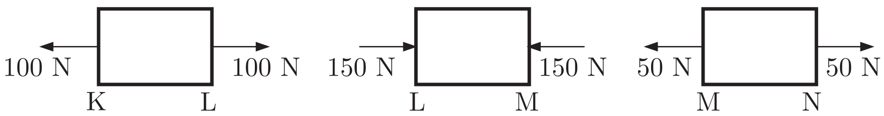
Since MN, NO and OP are connected in series combination. So angular deflection between the ends M and P of the shaft is,

$$\theta_{MP} = \theta_{MN} + \theta_{NO} + \theta_{OP} = \frac{10}{20} + \frac{10}{30} + \frac{10}{60} = 1\,\text{radian}$$

SOL 2.82 Option (B) is correct.

Given : $A = 25\,\text{mm}^2$, $E_{steel} = 200\,\text{GPa} = 200 \times 10^9\,\text{N/m}^2 = 200 \times 10^3\,\text{N/mm}^2$

First of all we have to make the F.B.D of the sections KL, LM and MN separately.



Now, From the F.B.D,

$$P_{KL} = 100\text{ N (Tensile)}$$

$$P_{LM} = -150\text{ N (Compressive)}$$

$$P_{MN} = 50\text{ N (Tensile)}$$

or $L_{KL} = 500\text{ mm}$, $L_{LM} =$ mm, $L_{MN} =$ mm

Total change in length,

$$\Delta L = \Delta L_{KL} + \Delta L_{LM} + \Delta L_{MN}$$

$$= \frac{P_{KL}L_{KL}}{AE} + \frac{P_{LM}L_{LM}}{AE} + \frac{P_{MN}L_{MN}}{AE} \qquad \Delta L = \frac{PL}{AE}$$

Substitute the values, we get

$$\Delta L = \frac{1}{25 \times 200 \times 10^3}[100 \times 500 - 150 \times 800 + 50 \times 400]$$

$$= \frac{1}{5000 \times 10^3}[-50000] = -10\,\mu\text{m}$$

SOL 2.82 Option (A) is correct.

Given : $d = 60\text{ mm}$, $T = 1600\text{ N–m}$

From the torsional formula,

$$\frac{T}{J} = \frac{\tau}{r} \qquad r = \frac{d}{\ } \text{ and } J = \frac{\pi}{\ } d$$

So, $$\tau_{\max} = \frac{T}{\frac{\pi}{32}d^4} \times \frac{d}{2} = \frac{16T}{\pi d^3}$$

Substitute the values, we get

$$\tau_{\max} = \frac{16 \times 1600}{3.14 \times (60 \times 10^{-3})^3} = \frac{8152.866}{(60)^3} \times 10^9$$

$$= 0.03774 \times 10^9\text{ Pa} = 37.74\text{ MPa} \simeq 37.72\text{ MPa}$$

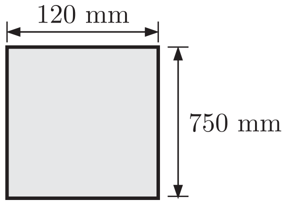
SOL 2.82 Option (A) is correct.

Given : $b = 120\text{ mm}$, $h = 750\text{ mm}$, $E_{steel} =$ G a $= 200 \times 10^3\text{ N/mm}^2$,
$W =$ kN m, $L =$ m

It is a uniformly distributed load. For a uniformly distributed load, maximum bending moment at centre is given by,

$$B.M. = \frac{WL^2}{8} = \frac{120 \times 15 \times 15}{8} = 3375\text{ kN-m}$$

SOL 2.82 Option (A) is correct.



We know that maximum deflection at the centre of uniformly distributed load is given by,

$$\delta_{\max} = \frac{5}{384} \times \frac{WL^4}{EI}$$

For rectangular cross-section,

$$I = \frac{bh^3}{12} = \frac{(120) \times (750)^3}{12} = 4.21875 \times 10^9\ \text{mm}^4 = 4.21875 \times 10^{-3}\ \text{m}^4$$

So,

$$\delta_{max} = \frac{5}{384} \times \frac{120 \times 10^3 \times (15)^4}{200 \times 10^9 \times 4.21875 \times 10^{-3}}$$

$$= \frac{5}{384} \times 7200 \times 10^{-3} = 0.09375\ \text{m} = 93.75\ \text{mm}$$

SOL 2.82 Option (D) is correct.

We know that, moment of inertia is defined as the second moment of a plane area about an axis perpendicular to the area.

Polar moment of inertia perpendicular to the plane of paper,

$$J \text{ or } I_P = \frac{\pi D^4}{32}$$

By the "perpendicular axis" theorem,

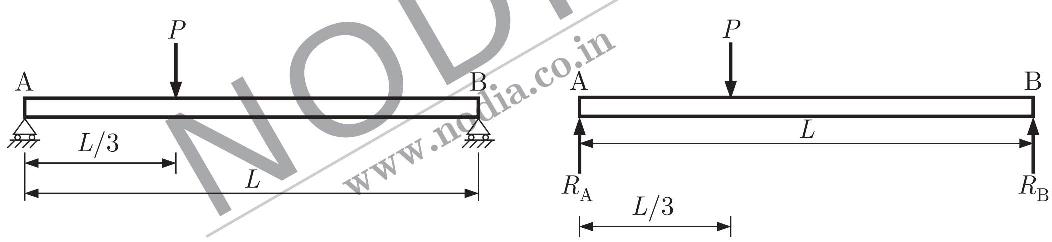
$$I_{XX} + I_{YY} = I_P \qquad \text{For circular section } I_{XX} = I_{YY}$$

$$2I_{XX} = I_P$$

$$I_{XX} = \frac{I_P}{2} = \frac{\pi D^4}{64} = I_{YY}$$

SOL 2.82 Option (D) is correct.

We know that, the simplest form of the simply supported beams is the beam supported on rollers at ends. The simply supported beam and the *FBD* shown in the Figure.



Where, R_A and R_B are the reactions acting at the ends of the beam.

In equilibrium condition of forces,

$$P = R_A + R_B \qquad \text{...(i)}$$

Taking the moment about point A,

$$R_B \times L = P \times \frac{L}{}$$

$$R_B = \frac{P}{3}$$

From equation (i),

$$R_A = P - R_B = P - \frac{P}{} = \frac{2P}{3}$$

Now bending moment at the point of application of the load

$$M = R_A \times \frac{L}{} = \frac{2P}{3} \times \frac{L}{3} = \frac{2PL}{9}$$

Or,

$$M = R_B \times \frac{L}{} = \frac{PL}{}$$

SOL 2.82 Option (C) is correct.

Given : $L_s = L_i$, $E_s = 206\ \text{GPa}$, $E_i = 100\ \text{GPa}$, $P_s = P_i$, $D_s = D_i$, $\Rightarrow A_s = A_i$

Where subscript s is for steel and i is for iron rod.

We know that elongation is given by,

$$\Delta L = \frac{PL}{AE}$$

Now, for steel or iron rod

$$\frac{\Delta L_s}{\Delta L_i} = \frac{P_s L_s}{A_s E_s} \times \frac{A_i E_i}{P_i L_i} = \frac{E_i}{E_s}$$

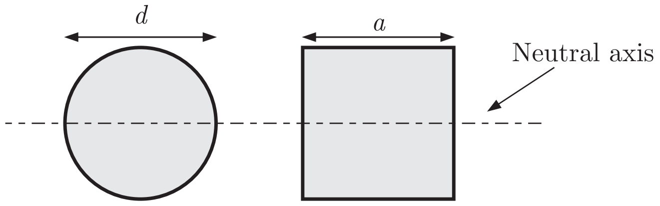
Substitute the values, we get

$$\frac{\Delta L_s}{\Delta L_i} = \frac{100}{206} = 0.485 < 1$$

or, $\Delta L_s < \Delta L_i \quad \Rightarrow \quad \Delta L_i > \Delta L_s$

So, cast iron rod elongates more than the mild steel rod.

SOL 2.82 Option (B) is correct.



Let, a = Side of square cross-section

d = diameter of circular cross-section

Using subscripts for the square and c for the circular cross section.

Given : $M_s = M_c;\ A_c = A_s$

So, $\frac{\pi}{4} d^2 = a$ ...(i)

From the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \quad \Rightarrow \quad \sigma = \frac{M}{I} \times y$$

Where, y = Distance from the neutral axis to the external fibre.

σ =Bending stress

For square cross-section bending stress,

$$\sigma_s = \frac{M_s}{\frac{a^4}{12}} \times \frac{a}{2} = \frac{M_s}{a} \quad \text{...(ii)}$$

And for circular cross-section,

$$\sigma_c = \frac{M_c}{\frac{\pi}{64} d^4} \times \frac{d}{2} = \frac{M_c}{d} \quad \text{...(iii)}$$

On dividing equation (iii) by equation (ii), we get

$$\frac{\sigma_c}{\sigma_s} = \frac{M_c}{d} \times \frac{a}{M_s} = \frac{16}{3}\frac{a^3}{d^3} \qquad M_c = M_s \text{ ...(iv)}$$

From equation (i),

$$\left(\frac{\pi}{4} d^2\right)^{3/2} = (a^2)^{3/2} = a$$

$$\frac{a}{d} = \left(\frac{\pi}{4}\right)^{3/2} = 0.695$$

Substitute this value in equation (iv), we get

$$\frac{\sigma_c}{\sigma_s} = \frac{16}{3} \times 0.695 = 3.706$$

$$\frac{\sigma_c}{\sigma_s} > 1 \quad \Rightarrow \quad \sigma_c > \sigma_s$$

So, Circular beam experience more bending stress than the square section.

SOL 2.82 Option (B) is correct.

Here k and k are in series combination and k is in parallel combination with this series combination.

So, $$k_{eq} = \frac{k \times k}{k + k} + k = \frac{k\,k + k\,k + k\,k}{k + k}$$

Natural frequency of the torsional oscillation of the disc, $\omega_n = \sqrt{\frac{k_{eq}}{J}}$

Substitute the value of k_{eq}, we get $$\omega_n = \sqrt{\frac{k\,k + k\,k + k\,k}{J(k + k)}}$$

SOL 2.82 Option (C) is correct.

Given : $\tau_1 = \tau_{\max} = 240\,\text{MPa}$

Let, diameter of solid shaft $d = d$, And Final diameter $d = 2d$ (Given)

From the Torsional Formula,

$$\frac{T}{J} = \frac{\tau}{r} \qquad \Rightarrow \quad T = \frac{\tau}{r} \times J$$

where, $J =$ polar moment of inertia. Given that torque is same,

$$\frac{\tau}{r} \times J = \frac{\tau}{r} \times J$$

$$\frac{\tau}{d} \times J = \frac{\tau}{d} \times J \qquad\qquad J = \frac{\pi}{32} d^4$$

$$\frac{\tau}{d} \times \frac{\pi}{\ } d = \frac{\tau}{d} \times \frac{\pi}{\ } d$$

$$\tau_1 \times d_1^3 = \tau_2 \times d_2^3 \quad \Rightarrow \quad \tau_2 = \tau_1 \times \frac{d_1^3}{d_2^3}$$

Substitute the values, we get

$$\tau_2 = 240 \times \left(\frac{d}{2d}\right)^3 = 240 \times \frac{1}{8} = 30\,\text{MPa}$$

Alternative Method :

From the Torsional Formula,

$$\tau = \frac{T\,r}{J} \qquad\qquad r = \frac{d}{\ } \text{ and } J = \frac{\pi}{\ } d$$

So, maximum shear stress,

$$\tau_{\max} = \frac{T}{\pi d} = 240\text{ MPa}$$

Given Torque is same and Shaft diameter is doubled then,

$$\tau'_{\max} = \frac{16\,T}{\pi\,(2d)^3} = \frac{16\,T}{8\pi d^3}$$

$$= \frac{\tau_{\max}}{8} = \frac{240}{8} = 30\,\text{MPa}$$

SOL 2.82 Option (A) is correct.

We know, differential equation of flexure for the beam is,

$$EI\frac{d^2 y}{dx^2} = M \qquad \Rightarrow \frac{d^2 y}{dx^2} = \frac{M}{EI}$$

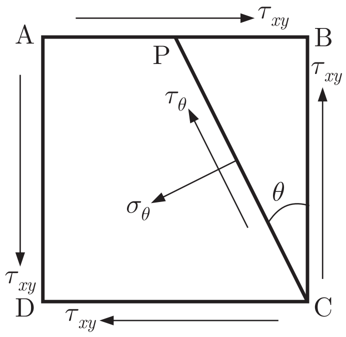
Integrating both sides, $$\frac{dy}{dx} = \frac{\ }{EI}\int M dx = \frac{\ }{EI} Mx + c$$

Again integrating, $$y = \frac{\ }{EI}\left(\frac{Mx^2}{2}\right) + c\,x + c_2 \qquad ...(i)$$

where, y gives the deflection at the given point. It is easily shown from the equation (i), If we increase the value of E and I, then deflection reduces.

SOL 2.82 Option (D) is correct.

Given figure shows stresses on an element subjected to pure shear.



Let consider a element to which shear stress have been applied to the sides AB and DC. Complementary stress of equal value but of opposite effect are then setup on sides AD and BC in order to prevent rotation of the element. So, applied and complementary shears are represented by symbol τ_{xy}.

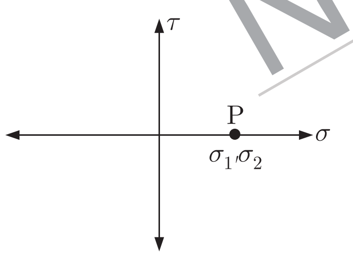
Consider the equilibrium of portion PBC. Resolving normal to PC assuming unit depth.

$$\begin{aligned}\sigma_\theta \times PC &= \tau_{xy} \times BC\sin\theta + \tau_{xy} \times PB\cos\theta \\ &= \tau_{xy} \times PC\cos\theta + \tau_{xy} \times PC\sin\theta\cos\theta \\ &= \tau_{xy}(\ \ \sin\theta\cos\theta) \times PC \\ \sigma_\theta &= 2\tau_{xy}\sin\theta\cos\theta\end{aligned}$$

The maximum value of σ_θ is τ_{xy} when $\theta = 45^\circ$.

$$\sigma_\theta = 2\tau\sin 45^\circ\cos 45^\circ \qquad \text{Given } (\tau_{xy} = \tau)$$

SOL 2.82 Option (B) is correct.



Given, Mohr's circle is a point located at 175 MPa on the positive Normal stress (at point P)

So, $\sigma_1 = \sigma_2 = 175\,\text{MPa}$, and $\tau_{\max} = 0$

So, both maximum and minimum principal stresses are equal.

Alternate Method :

$$\sigma_x = 175\,\text{MPa} \quad \sigma_y = \qquad \text{M} \ \ \text{a and } \tau_{xy} =$$

Maximum principal stress

$$\sigma_1 = \frac{1}{2}\Big[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y) + 4\tau_{xy}^2}\Big] = \frac{1}{2}[(175 + 175) + 0] = 175\,\text{MPa}$$

Minimum principal stress

$$\sigma_2 = \frac{1}{2}\Big[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y) + 4\tau_{xy}^2}\Big] = \frac{1}{2}[(175 + 175) - 0] = 175\,\text{MPa}$$

SOL 2.82 Option (D) is correct.

Mohr's circle is a point, and a point will move in every direction. So, the directions of maximum and minimum principal stresses at point P is in all directions. Every value of θ will give the same result of 175 MPa in all directions.

SOL 2.82 Option (D) is correct.

Mild steel is ductile in nature and it elongates appreciable before fracture.

The stress-strain curve of a specimen tested upto failure under tension is a measure of toughness.

SOL 2.82 Option (C) is correct.

3 dimensional stress tensor is defined as

$$z_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

There are 9 components of the stress tensor. But due to complementary nature of shear stresses,

$$\tau_{xy} = \tau_{yx},\ \tau_{xz} = \tau_{zx} \text{ and } \tau_{yz} = \tau_{zy}$$

So, we can say that the number of components in a stress tensor for defining stress at a point is 6 i.e. σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, τ_{zx}.

SOL 2.82 Option (A) is correct.

We know the volumetric strain is, $\varepsilon_\nu = \frac{(\ - \upsilon)}{E}(\sigma \ + \sigma \ + \sigma_3)$

Put $\sigma_1 = \sigma_2 = \sigma_3 = -\sigma$,

$$\varepsilon_\nu = \frac{\ - \upsilon}{E}(-3\sigma) = \frac{3(\ - \upsilon)}{E}\sigma \qquad \text{(in magnitude)}$$

The above equation gives the volumetric strain when the elemental volume is subjected to a compressive stress of σ from all sides. Negative sign indicates a compressive volumetric strain.

So, $\frac{\varepsilon_\nu}{\sigma} = \frac{3(\ - \upsilon)}{E} \Rightarrow \frac{\sigma}{\varepsilon_\nu} = \frac{E}{3(1-2\upsilon)}$

But $\frac{\sigma}{\varepsilon_\nu} = K$ (Bulk modulus)

Hence, $E = 3K(1-2\upsilon)$

SOL 2.82 Option (B) is correct.

According to Euler's theory, the crippling or buckling load (W_{cr}) under various end conditions is given by,

$$W_{cr} = \frac{C\pi \ EA}{L}$$

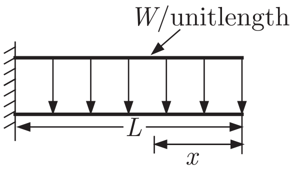
Where $C =$ constant, representing the end conditions of the column.

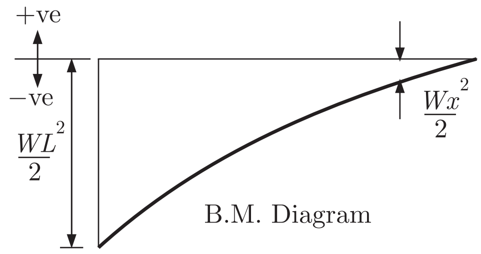
All parameters are same. So, $W_{cr} \propto C$

(i) For both ends fixed, $C =$

(ii) For both ends hinged, $C =$, so, $\frac{W_{(i)}}{W(ii)} = \frac{4}{1} = 4$

SOL 2.82 Option (D) is correct.

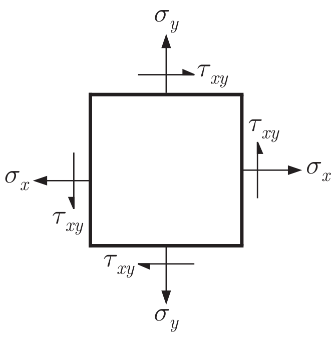




$$M_x = - Wx \times \frac{x}{} = -\frac{Wx}{}$$

The equation for M_x gives parabolic variations for $B\,M$ Maximum $B\,M$ occurs at $x = L$ and is equal to WL . (in magnitude)

SOL 2.82 Option (B) is correct.



For stress state the maximum principal stress is given by,

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

Here $\sigma_x = \sigma$, $\sigma_y = \sigma$ and $z_{xy} = \sigma$

Hence, $$\sigma_1 = \frac{1}{2}\left[(\sigma + \sigma) + \sqrt{0 + 4\sigma^2}\right] = \frac{1}{2}[2\sigma + 2\sigma] = 2\sigma$$

****